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Chargino and neutralino decays revisited

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Abstract. We perform a comprehensive analysis of the decays of charginos and neutralinos in the minimal supersymmetric standard model where the neutralino χ_1^0 is assumed to be the lightest supersymmetric particle. We focus, in particular, on the three-body decays of the next-to-lightest neutralino and the lightest chargino into the lightest neutralino and fermion–antifermion pairs and include vector boson, Higgs boson and sfermion exchange diagrams, where in the latter contribution the full mixing in the third generation is included. The radiative corrections to the heavy fermion and SUSY particle masses will also be taken into account. We present complete analytical formulae for the Dalitz densities and the integrated partial decay widths in the massless fermion case, as well as the expressions of the differential decay widths including the masses of the final fermions and the polarization of the decaying charginos and neutralinos. We then discuss these decay modes, in particular in scenarios where the parameter $\tan\beta$ is large and in models without universal gaugino masses at the grand unification scale, where some new decay channels, such as decays into gluinos and $q\bar{q}$ pairs, open up.

1 Introduction

In the minimal supersymmetric standard model (MSSM) [1,2], the lightest neutralinos χ_1^0, χ_2^0 and chargino $\chi_1^\pm$, which are mixtures of the higgsinos and gauginos that are the spin 1/2 partners of the Higgs and gauge bosons, are expected to be the lightest supersymmetric particles. In particular, the neutralino χ_1^0 is the lightest SUSY particle (LSP), which because of R-parity conservation [3], is stable and invisible. In models where the gaugino masses are unified at the grand unification scale [4], the masses of these particles are such that $m_{\chi_2^0} \sim m_{\chi_1^\pm} \sim 2m_{\chi_1^0}$ in the case where they are gaugino-like or $m_{\chi_2^0} \sim m_{\chi_1^\pm} \sim m_{\chi_1^0}$ in the case where they are higgsino-like. Thus, the states χ_2^0 and χ_1^+ are not much heavier than the LSP and might be the first SUSY particles to be discovered. The search for these sparticles is a major goal of present and future colliders, and the detailed study of their production and decay properties is mandatory in order to reconstruct the SUSY Lagrangian at the low-energy scale and to derive the structure of the theory at the high scale.

The decays of charginos and neutralinos have been widely discussed in the literature [5]. If the mass splitting between the LSP and the next-to-lightest neutralino χ_2^0 or the lightest chargino $\chi_1^\pm$ is larger than M_Z or M_W, the particles will decay into massive gauge bosons and the neutralino χ_1^0. If not, the decays will occur through virtual gauge boson and scalar fermion exchanges, leading in the final state to the LSP neutralino and a fermion–antifermion pair. Recently, it has been realized [6–9] that for large values of the parameter $\tan\beta$, the ratio of the vacuum expectation values of the two doublet Higgs fields which are needed to break the electroweak symmetry in the MSSM, the Yukawa couplings of third generation down-type fermions (b-quarks and τ-leptons), which are strongly enhanced, lead to dramatic consequences for the decays of these particles[1]. Indeed, the virtual exchanges of, on the one side, Higgs particles (because the Higgs boson couplings to b-quarks and τ-leptons are proportional to $\tan\beta$) and, on the other side, of third generation down-type sfermions (which tend to be lighter than the other sfermions in this case) become very important.

Furthermore, some attention has recently been devoted to models where the gaugino masses are not unified at the GUT scale, as might be the case in a large class of four-dimensional string models [12] or in the so-called anomaly-mediated SUSY-breaking models [13]. As an example, two particular cases have been discussed in [14], where SUSY-breaking occurs via an F-term that is not an SU(5) singlet and in an orbifold string model. In these models the gaugino masses at the electroweak scale can be very different

[1] Note that the scenario with $\tan\beta \sim m_t/m_b$ is favored in models with Yukawa coupling unification at the GUT scale [10]. In addition, large $\tan\beta$ values, $\tan\beta \gtrsim 3$–8 depending on the details of the radiative corrections, are needed to maximize the lightest h-boson mass in the MSSM, to cope with the LEP2 experimental bound $M_h \gtrsim 113.5\,\text{GeV}$ [11] in the decoupling regime where the h boson is standard model-like

from the pattern mentioned above. In particular, the χ_2^0 and $\chi_1^\pm$ masses can be closer to the LSP mass in some of these models, favoring the occurrence of three-body decays of the light chargino and neutralino states (including some new channels such as $\chi_2^0 \to q\bar{q}+$ gluino final states), while possibly disfavoring final states with heavy fermions (such as $b\bar{b}$ final states) and therefore dramatically affecting the decay branching ratios.

In this paper, we perform a detailed investigation of the three-body decay modes of charginos and neutralinos in the MSSM, focusing on the scenarios with large values of $\tan\beta$ and with non-unified gaugino masses at the GUT scale. We will provide complete analytical formulae for the Dalitz densities of the decays (in terms of the energies of the two final state fermions) and for the fully integrated partial decay widths. Furthermore, we will take into account the polarization of the decaying particle, which is needed in order to obtain the full correlations between the initial state in the production of these particles and the final states in their decays. We will also include the dependence on the masses of the final state fermions to have a more accurate prediction for final states involving b-quarks and τ-leptons (especially in scenarios where the mass difference between the decaying particles and the LSP is not very large) and to treat properly the case of heavy top quark final states. An important ingredient of the analysis will be the inclusion of the effects of the radiative corrections to the heavy fermion and chargino/neutralino masses, which will be shown to have a large impact.

This work extends the recent analyses made in [6–8] for chargino and neutralino decays, and completes our analyses of the higher order decays of SUSY particles (sfermions, in particular stops and sbottoms, and gluinos) in the MSSM [9,15].

This paper is organized as follows. In the next section, we will summarize the main features of the chargino, neutralino, sfermion and Higgs sectors of the MSSM which will be needed in our analysis. In Sect. 3, we will display the analytical expressions of the (unpolarized) Dalitz densities and the integrated partial three-body decay widths for massless final state fermions. Section 4 will be devoted to our numerical analysis and a short conclusion will be given in Sect. 5. In the appendix, we present the complete formulae for the partial decay widths, including the finite mass of the fermion final states and the polarization of the decaying charginos and neutralinos.

2 SUSY particles masses and couplings

To fix our notation, we will summarize in this section the main features of the chargino, neutralino, sfermion and Higgs sectors of the MSSM. We will then give, for completeness, all the couplings of these SUSY particles (i.e. couplings of the neutralinos and charginos to gauge and Higgs bosons and their couplings to fermion–sfermion pairs) as well as the couplings of MSSM Higgs and gauge bosons to fermions, which will be needed later when evaluating the two-body and three-body partial decay widths.

2.1 Masses and mixing

2.1.1 The chargino and neutralino systems

The general chargino mass matrix, in terms of the wino mass parameter M_2, the higgsino mass parameter μ and $\tan\beta$, is given by [16]

$$\mathcal{M}_C = \begin{bmatrix} M_2 & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{bmatrix}, \tag{2.1}$$

where we use $s_\beta \equiv \sin\beta, c_\beta \equiv \cos\beta$, etc. It is diagonalized by two real matrices U and V,

$$U^* \mathcal{M}_C V^{-1} \to U = \mathcal{O}_-$$

and

$$V = \begin{cases} \mathcal{O}_+, & \text{if } \det\mathcal{M}_C > 0, \\ \sigma_3\mathcal{O}_+, & \text{if } \det\mathcal{M}_C < 0, \end{cases} \tag{2.2}$$

where σ_3 is the Pauli matrix to make the chargino masses positive and $\mathcal{O}_\pm$ are rotation matrices, with angles given by

$$\tan 2\theta_- = \frac{2\sqrt{2}M_W(M_2 c_\beta + \mu s_\beta)}{M_2^2 - \mu^2 - 2M_W^2 c_\beta},$$

$$\tan 2\theta_+ = \frac{2\sqrt{2}M_W(M_2 s_\beta + \mu c_\beta)}{M_2^2 - \mu^2 + 2M_W^2 c_\beta}. \tag{2.3}$$

This leads to the two chargino masses:

$$m^2_{\chi^\pm_{1,2}} = \frac{1}{2}\Big\{M_2^2 + \mu^2 + 2M_W^2 \mp \Big[(M_2^2-\mu^2)^2 + 4M_W^2(M_W^2 c_{2\beta}^2 + M_2^2 + \mu^2 + 2M_2\mu s_{2\beta})\Big]^{1/2}\Big\}. \tag{2.4}$$

In the limit $|\mu| \gg M_2, M_W$, the masses of the two charginos reduce to

$$m_{\chi_1^\pm} \simeq M_2 - \frac{M_W^2}{\mu^2}\left(M_2 + \mu s_{2\beta}\right),$$

$$m_{\chi_2^\pm} \simeq |\mu| + \frac{M_W^2}{\mu^2}\epsilon_\mu\left(M_2 s_{2\beta} + \mu\right), \tag{2.5}$$

where ϵ_μ is for the sign of μ. For $|\mu| \to \infty$, the lightest chargino corresponds to a pure wino state with mass $m_{\chi_1^\pm} \simeq M_2$, while the heavier chargino corresponds to a pure higgsino state with a mass $m_{\chi_2^\pm} = |\mu|$.

In the case of the neutralinos, the four-dimensional neutralino mass matrix depends on the same two mass parameters μ and M_2, if the GUT relation $M_1 = (5/3)\tan^2\theta_\mathrm{W}$, $M_2 \simeq (1/2)M_2$ [16] is used. In the $(-\mathrm{i}\tilde{B}, -\mathrm{i}\tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0)$ basis, it has the form $(c_W^2 = 1 - s_W^2 = M_W^2/M_Z^2)$

$$\mathcal{M}_N = \begin{bmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{bmatrix}. \tag{2.6}$$

It can be diagonalized analytically [17] by a single real matrix Z. The expressions of the masses $m_{\chi_i^0}$ are rather involved. In the limit of large $|\mu|$ values, they however simplify to [18]

$$\begin{aligned} m_{\chi_1^0} &\simeq M_1 - \frac{M_Z^2}{\mu^2}(M_1 + \mu s_{2\beta})\, s_W^2, \\ m_{\chi_2^0} &\simeq M_2 - \frac{M_Z^2}{\mu^2}(M_2 + \mu s_{2\beta})\, c_W^2, \\ m_{\chi_3^0} &\simeq |\mu| + \frac{1}{2}\frac{M_Z^2}{\mu^2}\epsilon_\mu(1 - s_{2\beta})\left(\mu + M_2 s_W^2 + M_1 c_W^2\right), \\ m_{\chi_4^0} &\simeq |\mu| + \frac{1}{2}\frac{M_Z^2}{\mu^2}\epsilon_\mu(1 + s_{2\beta}) \\ &\quad \times \left(\mu - M_2 s_W^2 - M_1 c_W^2\right). \end{aligned} \tag{2.7}$$

Again, for $|\mu| \to \infty$, two neutralinos are pure gaugino states with masses $m_{\chi_1^0} \simeq M_1$, $m_{\chi_2^0} = M_2$, while the two others are pure higgsino states, with masses $m_{\chi_3^0} \simeq m_{\chi_4^0} \simeq |\mu|$. The matrix elements of the diagonalizing matrix, Z_{ij} with $i, j = 1, ..., 4$, are given by

$$\begin{aligned} Z_{i1} &= \left[1 + \left(\frac{Z_{i2}}{Z_{i1}}\right)^2 + \left(\frac{Z_{i3}}{Z_{i1}}\right)^2 + \left(\frac{Z_{i4}}{Z_{i1}}\right)^2\right]^{-1/2} \\ \frac{Z_{i2}}{Z_{i1}} &= -\frac{1}{\tan\theta_W}\frac{M_1 - \epsilon_i m_{\chi_i^0}}{M_2 - \epsilon_i m_{\chi_i^0}} \\ \frac{Z_{i3}}{Z_{i1}} &= \Big\{\Big(\mu(M_1 - \epsilon_i m_{\chi_i^0})(M_2 - \epsilon_i m_{\chi_i^0}) \\ &\quad - M_Z^2 s_\beta c_\beta[(M_1 - M_2)c_W^2 + M_2 - \epsilon_i m_{\chi_i^0}]\Big) \\ &\quad \Big/\Big(M_Z(M_2 - \epsilon_i m_{\chi_i^0}) s_W[\mu c_\beta + \epsilon_i m_{\chi_i^0} s_\beta]\Big)\Big\} \\ \frac{Z_{i4}}{Z_{i1}} &= \Big\{\Big(-\epsilon_i m_{\chi_i^0}(M_1 - \epsilon_i m_{\chi_i^0})(M_2 - \epsilon_i m_{\chi_i^0}) \\ &\quad - M_Z^2 c_\beta^2[(M_1 - M_2)c_W^2 + M_2 - \epsilon_i m_{\chi_i^0}]\Big) \\ &\quad \Big/\Big(M_Z(M_2 - \epsilon_i m_{\chi_i^0}) s_W[\mu c_\beta + \epsilon_i m_{\chi_i^0} s_\beta]\Big)\Big\}, \end{aligned} \tag{2.8}$$

where ϵ_i is the sign of the ith eigenvalue of the neutralino mass matrix, which in the large $|\mu|$ limit are: $\epsilon_1 = \epsilon_2 = 1$ and $\epsilon_4 = -\epsilon_3 = \epsilon_\mu$. Note that we will often use the rotated Z_{ij} matrix elements:

$$\begin{aligned} Z'_{i1} &= Z_{i1}c_W + Z_{i2}s_W, \quad Z'_{i2} = -Z_{i1}s_W + Z_{i2}c_W, \\ Z'_{i3} &= Z_{i3}, \quad Z'_{i4} = Z_{i4} \quad . \end{aligned} \tag{2.9}$$

We will not only discuss the chargino and neutralino spectrum in mSUGRA-type models, where the gaugino masses are unified at the GUT scale $M_{\rm GUT}$, but also when the boundary conditions at this high scale are different. For illustration, we focus on two scenarios discussed in [14]:

(i) Models in which SUSY-breaking occurs via an F-term that is not SU(5) singlet but belongs to a representation which appears in the symmetric product of two adjoints: $(\mathbf{24}\otimes\mathbf{24})_{\rm sym} = \mathbf{1}\oplus\mathbf{24}\oplus\mathbf{75}\oplus\mathbf{200}$ (where only model **1** leads to the universal gaugino masses discussed previously).

Table 1. Relative gaugino masses at $M_{\rm GUT}(M_Z)$ in the F_Φ representations and the OII model

F_Φ	M_3	M_2	M_1
1	$1(\sim 6)$	$1(\sim 2)$	$1(\sim 1)$
24	$2(\sim 12)$	$-3(\sim -6)$	$-1(\sim -1)$
75	$1(\sim 6)$	$3(\sim 6)$	$-5(\sim -5)$
200	$1(\sim 6)$	$2(\sim 4)$	$10(\sim 10)$
OII	$1(\sim 6)$	$5(\sim 10)$	$53/5(\sim 53/5)$

(ii) The **OII** model which is superstring motivated and where the SUSY-breaking is moduli-dominated.

The relation between the gaugino masses at the scale $M_{\rm GUT}$, $m_{1,2,3}$, and at the weak scale $\mathcal{O}(M_Z)$, $M_{1,2,3}$, are approximately given by the relation [19]:

$$M_1 \simeq 0.42 m_1, \quad M_2 \simeq 0.83 m_2, \quad M_3 \simeq 2.6 m_3, \tag{2.10}$$

leading to the well known hierarchy $M_1 : M_2 : M_3 = 1 : 2 : 6$ for a universal gaugino mass at the GUT scale, $m_1 = m_2 = m_3 = m_{1/2}$, as in mSUGRA-type models. The relative gaugino masses at $M_{\rm GUT}$ and at the low-energy scale M_Z are given in Table 1; see also [14]. The pattern for the neutralino and chargino masses can be quite different from the universal case **1**. In particular, for large values of the parameter μ, the LSP is wino-like in the scenario **200** where $M_2 < M_1$, implying that χ_1^0 and χ_1^+ are degenerate in mass. In the scenario **75**, the gauginos χ_1^0, χ_2^0 and χ_1^+ have masses which are very close since $|M_1| \sim |M_2|$, while in scenario **24**, the mass splitting between the LSP and the states χ_2^0, χ_1^+ can be very large. In the **OII** model and if no large loop corrections are present to increase the gluino mass compared to the value of $M_3 < M_1, M_2$ (to avoid the scenario with a gluino LSP), χ_1^0, χ_2^0 and χ_1^+ have to be higgsino-like and can be thus degenerate in mass.

Since χ_2^0 and χ_1^+ can be degenerate in mass with the LSP in some of these scenarios, it is important to include the radiative corrections to the masses. These corrections are quite involved [20]. Here we will work in two different approximations, which are valid in the (almost) pure gaugino and pure higgsino regions [21,22] and which reproduce the complete result to better than a few percent.

For gaugino-like neutralinos and charginos, $|\mu| \gg M_1, M_2, M_Z$, we will correct only the parameters M_1, M_2 in the chargino and neutralino mass matrices (which means that terms of $\mathcal{O}(\alpha/4\pi \times M_Z^2/\mu^2)$ are neglected); we assume that all fermions are massless and all squarks and sleptons are degenerate, with masses $m_{\tilde{q}}$ and $m_{\tilde{l}}$, respectively; furthermore, we work in the tree-level decoupling limit for the Higgs sector, where $M_h \sim M_Z$ and $M_H \sim M_{H^+} \sim M_A$ (see Sect. 2.1.3). For the gluino mass, $m_{\tilde{g}} = M_3 + \Delta M_3/M_3$, needed in order to compare to the LSP mass, we will include only the dominant QCD corrections.

In this limit, one then obtains for $\Delta M_{1,2,3}/M_{1,2,3}$ [21]:

$$\begin{aligned} \frac{\Delta M_1}{M_1} &= -\frac{\alpha}{4\pi c_W^2}\Big\{11 B_1(M_1^2, 0, m_{\tilde{q}}) + 9 B_1(M_1^2, 0, m_{\tilde{l}}) \\ &\quad - \frac{\mu}{M_1}s_{2\beta}[B_0(M_1^2, \mu, M_A) - B_0(M_1^2, \mu, M_Z)] \end{aligned}$$

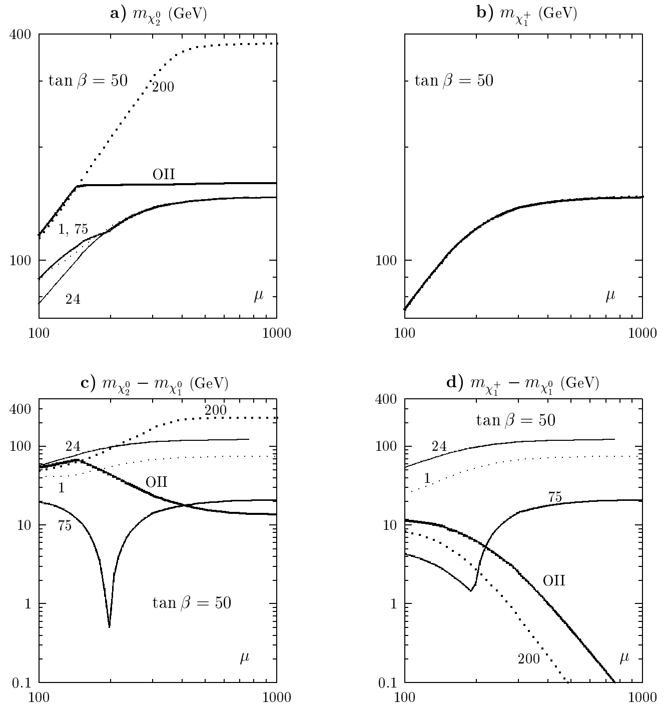

Fig. 1a–d. The masses of χ_2^0 and $\chi_1^\pm$ and their mass differences with the LSP χ_1^0 as a function of μ, for $\tan\beta = 50$ and $M_2 = 150\,\mathrm{GeV}$ with the M_1 values given in Table 1

$$+ B_1(M_1^2, \mu, M_A) + B_1(M_1^2, \mu, M_Z)\Big\}, \tag{2.11}$$

$$\begin{aligned}\frac{\Delta M_2}{M_2} &= -\frac{\alpha}{4\pi s_W^2}\Big\{9B_1(M_2^2, 0, m_{\tilde{q}}) + 3B_1(M_1^2, 0, m_{\tilde{l}}) \\ &- \frac{\mu}{M_2}s_{2\beta}[B_0(M_2^2, \mu, M_A) - B_0(M_2^2, \mu, M_Z)] \\ &+ B_1(M_2^2, \mu, M_A) + B_1(M_2^2, \mu, M_Z) \qquad (2.12) \\ &- 8B_0(M_2^2, M_2, M_W) \\ &+ 4B_1(M_2^2, M_2, M_W)\Big\},\end{aligned}$$

$$\begin{aligned}\frac{\Delta M_3}{M_3} &= \frac{3\alpha_s}{2\pi}\Big\{2B_0(M_3^2, M_3, 0) - B_1(M_3^2, M_3, 0) \\ &- 2B_1(M_3^2, 0, m_{\tilde{q}})\Big\}, \qquad (2.13)\end{aligned}$$

with the finite parts of the Passarino–Veltman two-point functions B_1 and B_0 given by [23]

$$\begin{aligned}&B_0(q^2, m_1, m_2) \\ &= -\mathrm{Log}\left(\frac{q^2}{Q^2}\right) - 2 - \mathrm{Log}(1 - x_+) - x_+\mathrm{Log}(1 - x_+^{-1}) \\ &\quad -\mathrm{Log}(1 - x_-) - x_-\mathrm{Log}(1 - x_-^{-1}) \\ &B_1(q^2, m_1, m_2) \\ &= \frac{1}{2q^2}\left[m_2^2\left(1 - \log\frac{m_2^2}{Q^2}\right) - m_1^2\left(1 - \mathrm{Log}\frac{m_1^2}{Q^2}\right)\right. \\ &\quad \left. +(q^2 - m_2^2 + m_1^2)B_0(q^2, m_1, m_2)\right], \qquad (2.14)\end{aligned}$$

with Q^2 denoting the renormalization scale and

$$x_\pm = \frac{1}{2q^2}\left(q^2 - m_2^2 + m_1^2 \pm \sqrt{(q^2 - m_2^2 + m_1^2)^2 - 4q^2(m_1^2 - \mathrm{i}\epsilon)}\right). \tag{2.15}$$

For higgsino-like χ_1^0, χ_2^0 and χ_1^+ particles, $|\mu| \ll M_{1,2}$, we will follow the approach of [22] and only correct the higgsino entries in the neutralino mass matrix and include the dominant Yukawa corrections to the light chargino and neutralino masses, due to stop/top and sbottom/bottom loops[2]. The masses in the higgsino limit [22] are then given by (we keep the sign of the eigenvalues):

$$\begin{aligned}m_{\chi_1^\pm} &\simeq |\mu + \delta_C|\left[1 - \frac{M_W^2 s_{2\beta}}{M_2(\mu + \delta_C)}\right] \qquad (2.16) \\ m_{\chi_{1,2}^0} &\simeq \mp(\mu + \delta_C) - \frac{M_Z^2}{2}(1 \mp s_{2\beta})\left(\frac{s_W^2}{M_1^2} + \frac{c_W^2}{M_2^2}\right) + \delta_N,\end{aligned}$$

with

$$\begin{aligned}\delta_C &= \frac{-3\alpha\mu}{8\pi}\left[\lambda_t^2\left(B_1(\mu^2, m_t, m_{\tilde{t}_1}) + B_1(\mu^2, m_t, m_{\tilde{t}_2})\right)\right. \\ &\quad \left. + \lambda_b^2\left(B_1(\mu^2, m_b, m_{\tilde{b}_1}) + B_1(\mu^2, m_b, m_{\tilde{b}_2})\right)\right], \\ \delta_N &= \frac{-3\alpha}{8\pi}\left[\lambda_t^2 m_t s_{2\theta_t}\left(B_0(\mu^2, m_t, m_{\tilde{t}_1}) - B_0(\mu^2, m_t, m_{\tilde{t}_2})\right)\right. \\ &\quad \left. + \lambda_b^2 m_b s_{2\theta_b}\left(B_0(\mu^2, m_b, m_{\tilde{b}_1}) - B_0(\mu^2, m_b, m_{\tilde{b}_2})\right)\right],\end{aligned} \tag{2.17}$$

where $\theta_{t,b}$ are the mixing angles in the stop and sbottom sectors (to be discussed in the next subsection) and $\lambda_{t,b}$ are the reduced Yukawa couplings of the t, b-quarks, which in terms of the running masses (also to be discussed in the next subsection) are given by

$$\lambda_b = \frac{m_b}{\sqrt{2}M_W s_W c_\beta}, \quad \lambda_t = \frac{m_t}{\sqrt{2}M_W s_W s_\beta}. \tag{2.18}$$

The $\chi_1^\pm$ and χ_2^0 masses as well as the mass differences $m_{\chi_1^\pm} - m_{\chi_1^0}$ and $m_{\chi_2^0} - m_{\chi_1^0}$ are shown in Fig. 1 as a function of μ for $\tan\beta = 50$, in the five models discussed above. The wino mass parameter is fixed at $M_2 = 150\,\mathrm{GeV}$ and the parameter M_1 is obtained from M_2 as in Table 1. We see that the mass difference between the lightest chargino and the LSP can be very small[3] in models **OII** and **200**, even after the inclusion of the radiative corrections. In model **75**, the next-to-lightest neutralino and the lightest

[2] We will further approximate the δ_C correction by δ_{34} in [22], which would be the case for almost degenerate squarks; the difference is negligible in general

[3] The search for charginos and neutralinos, which are almost degenerate in mass with the LSP, can be done in e^+e^- collisions, either via a search of almost stable particles or by a search of multi-pion final states with a large amount of missing energy; see for instance [24]. At hadron colliders, the direct search of such states will be very difficult, if possible at all

chargino can be degenerate in mass with the LSP for small values of μ, and the mass difference hardly exceeds 20 GeV (for the chosen value of M_2) even for large μ values. Note that for values $\mu \gtrsim M_3$, the gluino is lighter than the lightest neutralino χ_1^0 in model **OII**.

2.1.2 The sfermion system

The sfermion system is described, in addition to $\tan\beta$ and μ, by three parameters for each sfermion species: the left- and right-handed soft SUSY-breaking scalar masses $m_{\tilde{f}_{\rm L}}$ and $m_{\tilde{f}_{\rm R}}$ and the trilinear couplings A_f. In the case of the third generation scalar fermions, the mixing between left- and right-handed sfermions, which is proportional to the mass of the partner fermion, must be included [25]. The sfermion mass matrices read

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} m_f^2 + m_{\rm LL}^2 & m_f \tilde{A}_f \\ m_f \tilde{A}_f & m_f^2 + m_{\rm RR}^2 \end{pmatrix},$$

with

$$\begin{aligned} m_{\rm LL}^2 &= m_{\tilde{f}_{\rm L}}^2 + (I_3^f - e_f s_W^2) M_Z^2 c_{2\beta}, \\ m_{\rm RR}^2 &= m_{\tilde{f}_{\rm R}}^2 + e_f s_W^2 M_Z^2 c_{2\beta}, \\ \tilde{A}_f &= A_f - \mu (\tan\beta)^{-2I_3^f}, \end{aligned} \quad (2.19)$$

where I_3^f and e_f are the weak isospin and electric charge of the sfermion $\tilde{f}$, and $s_W^2 = 1 - c_W^2 \equiv \sin^2\theta_{\rm W}$. They are diagonalized by 2×2 rotation matrices of angle θ_f, which turn the current eigenstates, $\tilde{f}_{\rm L}$ and $\tilde{f}_{\rm R}$, into the mass eigenstates $\tilde{f}_1$ and $\tilde{f}_2$; the mixing angle and sfermion masses are then given by

$$\sin 2\theta_f = \frac{2 m_f \tilde{A}_f}{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2}, \quad \cos 2\theta_t = \frac{m_{\rm LL}^2 - m_{\rm RR}^2}{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2}, \quad (2.20)$$

$$m_{\tilde{f}_{1,2}}^2 = m_f^2 + \frac{1}{2}\left[m_{\rm LL}^2 + m_{\rm RR}^2 \mp \sqrt{(m_{\rm LL}^2 - m_{\rm RR}^2)^2 + 4 m_f^2 \tilde{A}_f^2} \right]. \quad (2.21)$$

The mixing is very strong in the stop sector for large values of $\tilde{A}_t$ and makes the lightest $\tilde{t}_1$ much lighter than the other squarks and possibly even lighter than the top quark itself. For large values of $\tan\beta$ and μ, the mixing in the sbottom and stau sectors can be also very strong, $\tilde{A}_{b,\tau} \sim -\mu\tan\beta$, leading to lighter $\tilde{b}_1$- and $\tilde{\tau}_1$-states.

Since the fermion masses provide one of the main inputs for sfermion mixing, it is important to include the leading radiative corrections to these parameters [26], in particular those due to strong interactions. The fermion masses which have to be used in the mass matrices (2.19) are the masses $\hat{m}_f(Q^2)$, evaluated in the $\overline{\rm DR}$ scheme at the scale Q and which, in terms of the pole masses m_f, are given by [21]

$$m_f = \hat{m}_f(Q^2)\left(1 + \frac{\Delta m_f}{m_f}\right). \quad (2.22)$$

In the case of top quarks, it is sufficient to include the one-loop QCD corrections originating from standard gluon exchange (first term) and gluino–stop exchange (second term):

$$\begin{aligned} \frac{\Delta m_t}{m_t} &= \frac{\alpha_s}{3\pi}\left[3\log\left(\frac{Q^2}{m_t^2}\right) + 5\right] \quad (2.23) \\ &- \frac{\alpha_s}{3\pi}\Big[B_1(m_{\tilde{g}}, m_{\tilde{t}_1}) + B_1(m_{\tilde{g}}, m_{\tilde{t}_2}) \\ &- s_{2\theta_t}\frac{m_{\tilde{g}}}{m_t}(B_0(m_{\tilde{g}}, m_{\tilde{t}_1}) - B_0(m_{\tilde{g}}, m_{\tilde{t}_2}))\Big], \end{aligned}$$

where in terms of $M = \max(m_1, m_2)$, $m = \min(m_1, m_2)$ and $x = m_2^2/m_1^2$, the two Passarino–Veltman functions [23] $B_{0,1}(m_1, m_2) \equiv B_{0,1}(0, m_1^2, m_2^2)$ simply read in this limit

$$\begin{aligned} B_0(m_1, m_2) &= -\log\left(\frac{M^2}{Q^2}\right) + 1 + \frac{m^2}{m^2 - M^2}\log\left(\frac{M^2}{m^2}\right) \\ B_1(m_1, m_2) &= \frac{1}{2}\left[-\log\left(\frac{M^2}{Q^2}\right) + \frac{1}{2} + \frac{1}{1-x} \right. \\ &\left. + \frac{\log x}{(1-x)^2} - \theta(1-x)\log x\right]. \end{aligned} \quad (2.24)$$

In the case of bottom quarks, the first important correction which has to be included is the one due to standard QCD corrections and the running from the scale m_b to the high scale Q. The $\overline{\rm DR}$ b-quark mass (for the NNLO corrections, we assume that the correction in the $\overline{\rm MS}$ and $\overline{\rm DR}$ schemes are the same, since the latter is not yet available) is given by [27]:

$$\hat{m}_b(Q^2) = \hat{m}_b(m_b^2)c[\alpha_s(Q^2)/\pi]/c[\alpha_s(m_b^2)/\pi], \quad (2.25)$$

with

$$\hat{m}_b(m_b^2) = m_b\left[1 + \frac{5}{3}\frac{\alpha_s(m_b^2)}{\pi} + 12.4\frac{\alpha_s^2(m_b^2)}{\pi^2}\right], \quad (2.26)$$

$$\begin{aligned} c(x) &= (23x/6)^{12/23}[1 + 1.175x + 1.5x^2], \\ &\quad \text{for } Q^2 < m_t^2, \\ c(x) &= (7x/2)^{4/7}[1 + 1.398x + 1.8x^2], \\ &\quad \text{for } Q^2 > m_t^2. \end{aligned} \quad (2.27)$$

After this, one has to include the sbottom–gluino and the stop–chargino corrections which are the most important ones [21], in particular for large $\tan\beta$ and μ values:

$$\begin{aligned} \frac{\Delta m_b}{m_b} &= -\frac{\alpha_s}{3\pi}\Big[B_1(m_{\tilde{g}}, m_{\tilde{b}_1}) + B_1(m_{\tilde{g}}, m_{\tilde{b}_2}) \\ &- s_{2\theta_b}\frac{m_{\tilde{g}}}{m_b}(B_0(m_{\tilde{g}}, m_{\tilde{b}_1}) - B_0(m_{\tilde{g}}, m_{\tilde{b}_2}))\Big] \\ &- \frac{\alpha}{8\pi s_W^2}\frac{m_t\mu}{M_W^2\sin 2\beta}s_{2\theta_t}[B_0(\mu, m_{\tilde{t}_1}) - B_0(\mu, m_{\tilde{t}_2})] \\ &- \frac{\alpha}{4\pi s_W^2}\left[\frac{M_2\mu\tan\beta}{\mu^2 - M_2^2}(c_{\theta_t}^2 B_0(M_2, m_{\tilde{t}_1}) \right. \\ &\left. + s_{\theta_t}^2 B_0(M_2, m_{\tilde{t}_2})) + (\mu \leftrightarrow M_2)\right]. \end{aligned} \quad (2.28)$$

For the τ-lepton mass, the only relevant corrections to be included are those stemming from chargino–sneutrino loops, and which simply read

$$\frac{\Delta m_\tau}{m_\tau} = -\frac{\alpha}{4\pi s_W^2}\frac{M_2\mu\tan\beta}{\mu^2 - M_2^2}\left[B_0(M_2, m_{\tilde{\nu}_\tau}) - B_0(\mu, m_{\tilde{\nu}_\tau})\right]. \tag{2.29}$$

The effect of the radiative corrections is shown in Fig. 2 for the case of the bottom quark and tau lepton masses for $\tan\beta = 50$ as a function of μ for the various models with and without unification of the gaugino masses at M_{GUT}. The wino mass is fixed at $M_2 = 150\,\mathrm{GeV}$ and $M_1, M_3 \simeq m_{\tilde{g}}$ at the weak scale are given in Table 1. The main correction to the $\overline{\mathrm{DR}}$ bottom quark mass, $\hat{m}_b(M_Z^2) \sim 3\,\mathrm{GeV}$, is due to the SUSY–QCD corrections from gluino–sbottom loops in the case of large values of $\tan\beta$ and μ. This correction is proportional to $\Delta m_b \sim -(\alpha_s/\pi) \times \tan\beta\mu m_{\tilde{g}}/m_{\tilde{b}}^2$ and can increase or decrease (depending on the sign of μ] the b-quark mass by more than a factor of two. The effect of the radiative corrections is less drastic in the case of the τ-mass since the latter are of the order a few percent.

Let us now discuss the dependence of the sfermion masses on the gaugino masses as well as on the parameters μ and $\tan\beta$, in models with a universal mass m_0 for the scalar fermions at the scale M_{GUT}, but without the gaugino mass unification assumption $m_{1,2,3} = m_{1/2}$. In the case of the partners of the light fermions (including b-quarks), one can neglect to a good approximation the effect of the Yukawa couplings in the one-loop renormalization group evolution of the soft SUSY-breaking scalar masses. With the notation of the first generation, one then obtains, when including the D-terms, the following expressions [19]:

$$\begin{aligned}
m^2_{\tilde{u}_\mathrm{L}} &= m_0^2 + 5.8m_3^2 + 0.47m_2^2 + 4.2\times10^{-3}m_1^2 \\
&\quad + 0.35M_Z^2\cos2\beta, \\
m^2_{\tilde{d}_\mathrm{L}} &= m_0^2 + 5.8m_3^2 + 0.47m_2^2 + 4.2\times10^{-3}m_1^2 \\
&\quad - 0.42M_Z^2\cos2\beta, \\
m^2_{\tilde{u}_\mathrm{R}} &= m_0^2 + 5.8m_3^2 + 6.6\times10^{-2}m_1^2 + 0.16M_Z^2\cos2\beta, \\
m^2_{\tilde{d}_\mathrm{R}} &= m_0^2 + 5.8m_3^2 + 1.7\times10^{-2}m_1^2 - 0.08M_Z^2\cos2\beta, \\
m^2_{\tilde{\nu}_\mathrm{L}} &= m_0^2 + 0.47m_2^2 + 3.7\times10^{-2}m_1^2 + 0.50M_Z^2\cos2\beta, \\
m^2_{\tilde{e}_\mathrm{L}} &= m_0^2 + 0.47m_2^2 + 3.7\times10^{-2}m_1^2 - 0.27M_Z^2\cos2\beta, \\
m^2_{\tilde{e}_\mathrm{R}} &= m_0^2 + 0.15m_1^2 - 0.23M_Z^2\cos2\beta.
\end{aligned} \tag{2.30}$$

One has then, in the case of sbottoms and staus, to include the mixing since in this case, large enough off-diagonal elements of the mass matrices are obtained for large μ and $\tan\beta$ values (the effect of the trilinear couplings plays only a marginal role).

The squark masses are governed by the parameter m_3, while the slepton masses are governed by the parameter m_2, and to a lesser extent m_1. Figures 3a,b show the variation of the soft parameters $m_{\tilde{b}_1}$ (a) and $m_{\tilde{\tau}_1}$ (b) as a function of M_2 for $\tan\beta = 50$ and $m_0 = 300\,\mathrm{GeV}$. As can be seen, depending on the models, the squark and slepton masses can be different for different models. In Fig. 3c, the masses $m_{\tilde{d}_\mathrm{R}}, m_{\tilde{e}_\mathrm{R}}$ and $m_{\tilde{b}_1}, m_{\tilde{\tau}_1}$ are shown as a function of μ for $\tan\beta = 50$; we have used the previous equations and fixed $m_0 = 300\,\mathrm{GeV}$ and $m_1 = m_2 = m_3 = m_{1/2} = 120\,\mathrm{GeV}$, i.e. as in the mSUGRA-type scenario. While for small values of μ, and hence small off-diagonal elements in the $\tilde{b}$ and $\tilde{\tau}$ mass matrices, $\tilde{d}_\mathrm{R}, \tilde{b}_1$ and $\tilde{e}_\mathrm{R}, \tilde{\tau}_1$ are almost degenerate in mass, the mass splitting increases with increasing μ reaching a substantial amount for $\mu \geq m_0$.

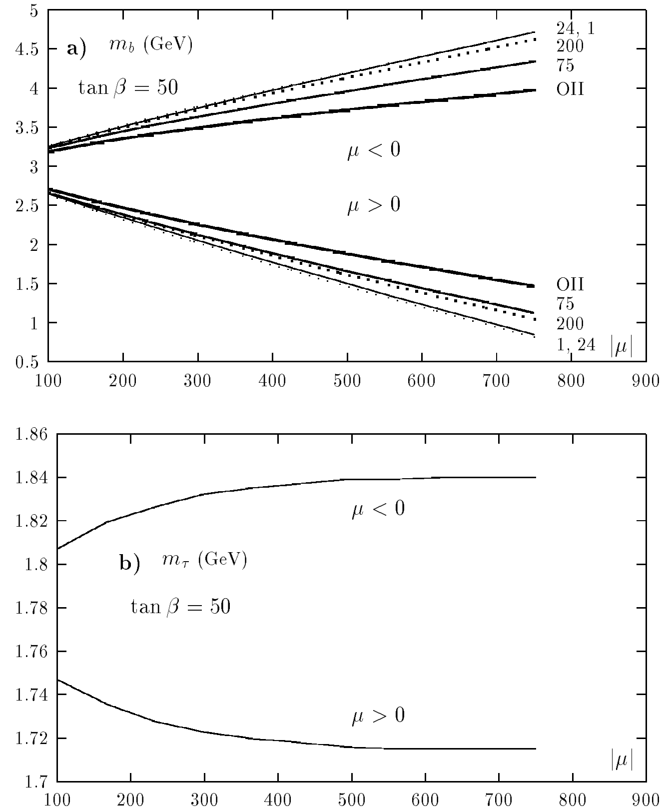

Fig. 2a,b. The b-quark and τ-lepton masses, including the radiative corrections, as a function of μ, for $\tan\beta = 50$ and $M_2 = 150\,\mathrm{GeV}$ in the various models of Table 1

2.1.3 The Higgs sector

The MSSM includes two iso-doublets of Higgs fields, which after spontaneous symmetry breaking, give rise to a quintet of physical Higgs boson states: h, H, A, $H^\pm$ [28]. While an upper bound of about 130 GeV can be derived on the mass of the light CP-even neutral Higgs boson h [29], the heavy CP-even and CP-odd neutral Higgs bosons H, A, and the charged Higgs bosons $H^\pm$ may have masses of the order of the electroweak symmetry scale v up to about 1 TeV. This extended Higgs system can be described by two parameters at the tree level: $\tan\beta$ and one mass parameter which is generally identified with the pseudoscalar mass M_A. The Higgs mass parameters and the couplings are affected by top and stop loop radiative corrections [29],

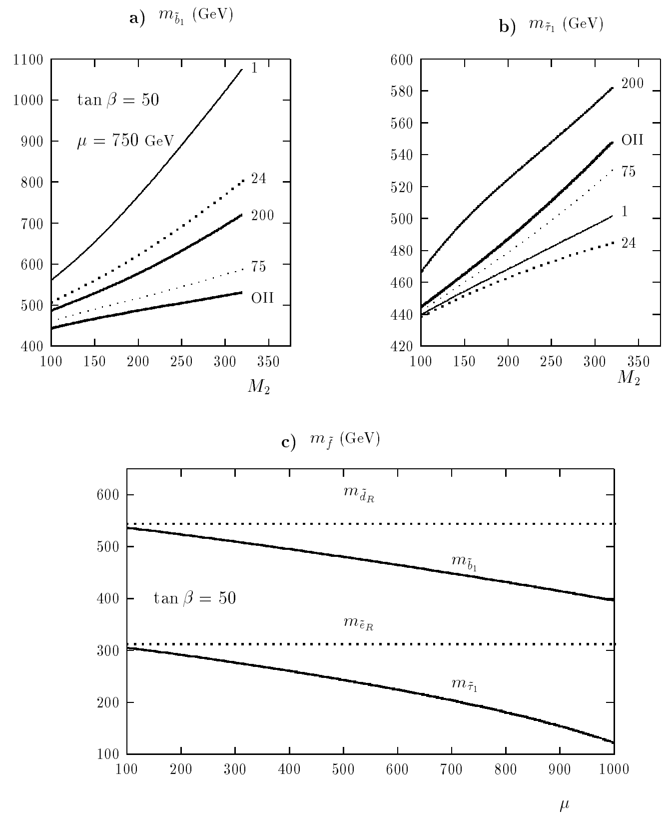

Fig. 3a–c. The masses of lightest sbottom **a** and tau slepton **b** as a function of M_2 for $m_0 = 300$ GeV and $\mu = 750$ GeV in the models of Table 1. The $\tilde{b}_1, \tilde{d}_{\rm R}$ and $\tilde{\tau}_1, \tilde{e}_{\rm R}$ masses as a function of μ, for $\tan\beta = 50$ and $M_2 = 150$ GeV in model 1, c

which in the leading approximation are parameterized by

$$\epsilon \approx \frac{3G_{\rm F} m_t^4}{\sqrt{2}\pi^2 \sin^2\beta} \log \frac{\tilde{m}^2}{m_t^2}, \tag{2.31}$$

where the scale of supersymmetry breaking is characterized by a common squark-mass value $\tilde{m}$. The next-to-leading order QCD corrections can be included by using the running top quark mass in the $\overline{\rm MS}$ scheme. Stop mixing effects can be accounted for by shifting $\tilde{m}^2$ in (2.31) by the amount ($\tilde{A}_t = A_t - \mu\cot\beta$)

$$\tilde{m}^2 \to \tilde{m}^2 + \Delta\tilde{m}^2 : \quad \Delta\tilde{m}^2 = \tilde{A}_t^2[1 - \tilde{A}_t^2/(12\tilde{m}^2)]. \tag{2.32}$$

The neutral CP-even and charged Higgs boson masses and the mixing angle α in the neutral sector, when expressed in terms of M_A and $\tan\beta$, are given in this approximation by

$$\begin{aligned} M_{h,H}^2 &= \frac{1}{2}\Big[M_A^2 + M_Z^2 + \epsilon \\ &\mp \sqrt{(M_A^2 + M_Z^2 + \epsilon)^2 - 4M_A^2 M_Z^2 c_{2\beta}^2 - 4\epsilon(M_A^2 s_\beta^2 + M_Z^2 c_\beta^2)}\Big], \\ M_{H^\pm}^2 &= M_W^2 + M_A^2, \\ \tan 2\alpha &= \tan 2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2 + \epsilon/c_{2\beta}}, \\ &\quad \text{with } -\frac{\pi}{2} \le \alpha \le 0. \end{aligned} \tag{2.33}$$

In the decoupling limit, $M_A \gg M_Z$, the $A, H, H^\pm$ bosons become degenerate in mass $M_A \simeq M_H \simeq M_{H^\pm}$ while the lightest h boson reaches its maximal mass value $M_h^2 \sim M_Z^2 + \epsilon$; the angle α approaches the value $\alpha \to \beta - \pi/2$. The couplings of the h particle to fermions and gauge bosons are then SM-like, while the couplings of the $H, A, H^\pm$ bosons to down (up)-type fermions are (inversely) proportional to $\tan\beta$.

In the present analysis, we will use the full renormalization-group improved radiative corrections to the Higgs sector given in [30]. We will often denote the Higgs bosons by H_k with $k = 1, 2, 3, 4$, corresponding to H, h, A and $H^\pm$, respectively.

2.2 Couplings

In this subsection, we list the various couplings [2,16,18] which will be needed in our analysis. All the couplings are normalized to the electric charge e.

(1) The couplings of the charginos and neutralinos to the weak gauge bosons $W^\pm, Z$:

$$\begin{aligned} G^{\rm L,R}_{\chi_i^0 \chi_j^+ W^+} &= G^{\rm L,R}_{ijW}, \quad \text{with} \\ G^{\rm L}_{ijW} &= \frac{1}{\sqrt{2}s_W}[-Z_{i4}V_{j2} + \sqrt{2}Z_{i2}V_{j1}], \\ G^{\rm R}_{ijW} &= \frac{1}{\sqrt{2}s_W}[Z_{i3}U_{j2} + \sqrt{2}Z_{i2}U_{j1}], \end{aligned} \tag{2.34}$$

$$\begin{aligned} G^{\rm L,R}_{\chi_i^- \chi_j^+ Z} &= G^{\rm L,R}_{ijZ}, \quad \text{with} \\ G^{\rm L}_{ijZ} &= \frac{1}{c_W s_W}\left[-\frac{1}{2}V_{i2}V_{j2} - V_{i1}V_{j1} + \delta_{ij}s_W^2\right], \\ G^{\rm R}_{ijZ} &= \frac{1}{c_W s_W}\left[-\frac{1}{2}U_{i2}U_{j2} - U_{i1}U_{j1} + \delta_{ij}s_W^2\right], \end{aligned} \tag{2.35}$$

$$\begin{aligned} G^{\rm L,R}_{\chi_i^0 \chi_j^0 Z} &= G^{\rm L,R}_{ijZ}, \quad \text{with} \\ G^{\rm L}_{ijZ} &= -\frac{1}{2s_W c_W}[Z_{i3}Z_{j3} - Z_{i4}Z_{j4}], \\ G^{\rm R}_{ijZ} &= +\frac{1}{2s_W c_W}[Z_{i3}Z_{j3} - Z_{i4}Z_{j4}]. \end{aligned} \tag{2.36}$$

(2) The couplings of charginos and neutralinos to the Higgs bosons:

$$\begin{aligned} G^{\rm L,R}_{\chi_i^0 \chi_j^+ H^+} &= G^{\rm L,R}_{ij4}, \quad \text{with} \\ G^{\rm L}_{ij4} &= \frac{c_\beta}{s_W}\left[Z_{j4}V_{i1} + \frac{1}{\sqrt{2}}\left(Z_{j2} + \tan\theta_{\rm W} Z_{j1}\right)V_{i2}\right], \\ G^{\rm R}_{ij4} &= \frac{s_\beta}{s_W}\left[Z_{j3}U_{i1} - \frac{1}{\sqrt{2}}\left(Z_{j2} + \tan\theta_{\rm W} Z_{j1}\right)U_{i2}\right], \end{aligned}$$

$$\begin{aligned} G^{\rm L,R}_{\chi_i^- \chi_j^+ H_k^0} &= G^{\rm L,R}_{ijk}, \quad \text{with} \\ G^{\rm L}_{ijk} &= \frac{1}{\sqrt{2}s_W}\left[e_k V_{j1}U_{i2} - d_k V_{j2}U_{i1}\right], \end{aligned}$$

$$G^{\rm R}_{ijk} = \frac{1}{\sqrt{2} s_W} \left[e_k V_{i1} U_{j2} - d_k V_{i2} U_{j1} \right] \epsilon_k, \qquad (2.37)$$

$$G^{\rm L,R}_{\chi^0_i \chi^+_j H^+} = G^{\rm LR}_{ij4}, \quad \text{with}$$

$$G^{\rm L}_{ijk} = \frac{1}{2 s_W} \left(Z_{j2} - \tan\theta_{\rm W} Z_{j1} \right) \times \left(e_k Z_{i3} + d_k Z_{i4} \right) + i \leftrightarrow j$$

$$G^{\rm R}_{ijk} = \frac{1}{2 s_W} \left(Z_{j2} - \tan\theta_{\rm W} Z_{j1} \right) \times \left(e_k Z_{i3} + d_k Z_{i4} \right) \epsilon_k + i \leftrightarrow j, \qquad (2.38)$$

where $\epsilon_{1,2} = -\epsilon_3 = 1$ and the coefficients e_k and d_k read

$$e_1/d_1 = c_\alpha / - s_\alpha, \quad e_2/d_2 = -s_\alpha / - c_\alpha, \quad e_3/d_3 = -s_\beta / c_\beta. \qquad (2.39)$$

(3) For the couplings between neutralinos, fermions and sfermions, $\tilde{f}_i - f - \chi^0_j$, one has

$$\left\{ \begin{array}{c} a^{\tilde{f}}_{j1} \\ a^{\tilde{f}}_{j2} \end{array} \right\} = - \frac{m_f r_f}{\sqrt{2} M_W s_W} \left\{ \begin{array}{c} s_{\theta_f} \\ c_{\theta_f} \end{array} \right\} - e^f_{{\rm L}j} \left\{ \begin{array}{c} c_{\theta_f} \\ -s_{\theta_f} \end{array} \right\},$$

$$\left\{ \begin{array}{c} b^{\tilde{f}}_{j1} \\ b^{\tilde{f}}_{j2} \end{array} \right\} = - \frac{m_f r_f}{\sqrt{2} M_W s_W} \left\{ \begin{array}{c} c_{\theta_f} \\ -s_{\theta_f} \end{array} \right\} - e^f_{{\rm R}j} \left\{ \begin{array}{c} s_{\theta_f} \\ c_{\theta_f} \end{array} \right\}, \qquad (2.40)$$

with $r_u = Z_{j4}/\sin\beta$ and $r_d = Z_{j3}/\cos\beta$ for up- and down-type fermions, and

$$e^f_{{\rm L}j} = \sqrt{2} \left[e_f Z'_{j1} + \left(I^3_f - e_f s^2_W \right) \frac{1}{c_W s_W} Z'_{j2} \right],$$

$$e^f_{{\rm R}j} = -\sqrt{2} e_f \left[Z'_{j1} - \frac{s_W}{c_W} Z'_{j2} \right]. \qquad (2.41)$$

(4) For the couplings between charginos, fermions and sfermions, $\tilde{f}_i - f' - \chi^+_j$, one has for up-type and down-type sfermions

$$\left\{ \begin{array}{c} a^{\tilde{u}}_{j1} \\ a^{\tilde{u}}_{j2} \end{array} \right\} = \frac{V_{j1}}{s_W} \left\{ \begin{array}{c} -c_{\theta_u} \\ s_{\theta_u} \end{array} \right\} + \frac{m_u V_{j2}}{\sqrt{2} M_W s_W s_\beta} \left\{ \begin{array}{c} s_{\theta_u} \\ c_{\theta_u} \end{array} \right\},$$

$$\left\{ \begin{array}{c} b^{\tilde{u}}_{j1} \\ b^{\tilde{u}}_{j2} \end{array} \right\} = \frac{m_d U_{j2}}{\sqrt{2} M_W s_W c_\beta} \left\{ \begin{array}{c} c_{\theta_u} \\ -s_{\theta_u} \end{array} \right\}, \qquad (2.42)$$

$$\left\{ \begin{array}{c} a^{\tilde{d}}_{j1} \\ a^{\tilde{d}}_{j2} \end{array} \right\} = \frac{U_{j1}}{s_W} \left\{ \begin{array}{c} -c_{\theta_d} \\ s_{\theta_d} \end{array} \right\} + \frac{m_d U_{j2}}{\sqrt{2} M_W s_W c_\beta} \left\{ \begin{array}{c} s_{\theta_d} \\ c_{\theta_d} \end{array} \right\},$$

$$\left\{ \begin{array}{c} b^{\tilde{d}}_{j1} \\ b^{\tilde{d}}_{j2} \end{array} \right\} = \frac{m_u V_{j2}}{\sqrt{2} M_W s_W s_\beta} \left\{ \begin{array}{c} c_{\theta_d} \\ -s_{\theta_d} \end{array} \right\}. \qquad (2.43)$$

(5) Finally, the couplings of the W, Z gauge bosons and the four Higgs bosons $H_k = H, h, A, H^\pm$ with $k = 1, ..., 4$ to fermions are

$$v^f_Z = \frac{2 I^3_f - 4 e_f s^2_W}{4 c_W s_W}, \quad a^f_Z = \frac{2 I^3_f}{4 c_W s_W},$$

$$v^f_W = a^f_W = \frac{1}{2\sqrt{2} s_W}, \qquad (2.44)$$

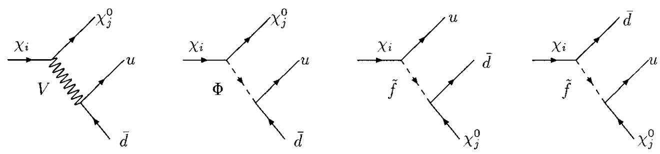

Fig. 4. The Feynman diagrams contributing to the three-body decays of charginos and neutralinos into the LSP and two fermions

$$v^f_1 = \frac{m_f r^f_2}{2 s_W M_W}, \quad a^f_1 = 0, \quad v^f_2 = \frac{m_f r^f_1}{2 s_W M_W}, \quad a^f_2 = 0,$$

$$v^f_3 = 0, \quad a^f_3 = \frac{-m_f (\tan\beta)^{-2 I^3_f}}{2 s_W M_W}, \qquad (2.45)$$

$$v^f_4 = - \frac{m_d \tan\beta + m_u \cot\beta}{2\sqrt{2} s_W M_W},$$

$$a^f_4 = \frac{m_d \tan\beta - m_u \cot\beta}{2\sqrt{2} s_W M_W}, \qquad (2.46)$$

with the coefficients $r^f_{1,2}$:

$$r^u_1 = s_\alpha / s_\beta, \quad r^u_2 = c_\alpha / s_\beta,$$

$$r^d_1 = c_\alpha / c_\beta, \quad r^d_2 = -s_\alpha / c_\beta. \qquad (2.47)$$

3 Three-body decays

In this section we give the complete analytical expressions of the partial widths of the three-body decays of charginos and neutralinos into a neutralino and two fermions, that we will denote to be general by u and $\bar{d}$ (although they can be the same)

$$\chi_i \to \chi^0_j u \bar{d}. \qquad (3.1)$$

We will not assume that the final neutralino is the LSP χ^0_1, but any of the neutralinos χ^0_j to cover also the possibility of cascade decays. As shown in Fig. 4, these decays proceed through gauge boson exchange ($V = W$ and Z for χ^+_i and χ^0_i decays, respectively), Higgs boson exchange ($H_k = H^+$ for χ^+_i decays and $H_k = H, h, A$ with $k = 1, 2, 3$ for χ^0_i decays) and sfermion exchange in the t- and u-channels (the flavor is fixed by the sfermion–fermion and final neutralino vertex). For gluino decays [31,9], only the channels with u- and t-channel squark exchange will be present; the partial widths can be straightforwardly derived from those of the neutralino decays, with the appropriate change of the couplings. Note that for the treatment of the Majorana nature of the initial state, we use the rules given in [32].

In this section, we will simply give the complete analytical expressions for the (unpolarized) Dalitz plot density in terms of the energies of two final fermions, and for the fully integrated partial widths for vanishing fermion masses[4]. (In the most complete analysis of these decays

[4] In mSUGRA-type models, this approximation is very good for all light fermion final states, including b-quarks and τ-

available in the literature up to now, [6], the fully integrated partial widths have not been derived: one integral has been left-out and performed numerically.) The formulae for the general case with non-vanishing values for the masses of the final standard fermions (to be able to describe more accurately the cases of chargino decays into $\tau\nu$ as well as neutralino decays into $b\bar{b}$ and $\tau^+\tau^-$ final states and to treat the case of the top quark) and where the polarization of the initial gauginos are taken into account, are given in the appendix.

3.1 The Dalitz densities for the three-body decays

The Dalitz density of the decay mode (3.1) is given in terms of the reduced energies of the two final state fermions

$$x_1 = 2E_u/m_{\chi_i}, \quad x_2 = 2E_d/m_{\chi_i}, \\ x_3 = 2E_{\chi_j}/m_{\chi_i} = 2 - x_1 - x_2, \tag{3.2}$$

but we will also use the simplifying notation:

$$y_1 = 1 - x_1 - \mu_\chi, \quad y_2 = 1 - x_2 - \mu_\chi, \\ y_3 = 1 - x_3 + \mu_\chi, \tag{3.3}$$

with the reduced masses $\mu_X^2 = M_X^2/m_{\chi_i}^2$ (for the final state neutralino we drop the index, i.e. $\mu_\chi = m^2_{\chi_j^0}/m^2_{\chi_i}$). Neglecting the masses of the final fermions (but not in the couplings) and the widths of the exchanged (s)particles, the Dalitz density is given by

$$\frac{\mathrm{d}\Gamma_{\chi_i}}{\mathrm{d}x_1 \mathrm{d}x_2} = \frac{e^4 m_{\chi_i}}{64(2\pi)^3} N_c \Big[\mathrm{d}\Gamma_V + \mathrm{d}\Gamma_{\tilde{u}} + \mathrm{d}\Gamma_{\tilde{d}} + \mathrm{d}\Gamma_\Phi \\ + \mathrm{d}\Gamma_{H_1H_2} + \mathrm{d}\Gamma_{V\tilde{u}} + \mathrm{d}\Gamma_{V\tilde{d}} \\ + \mathrm{d}\Gamma_{\tilde{u}\tilde{d}} + \mathrm{d}\Gamma_{\Phi\tilde{u}} + \mathrm{d}\Gamma_{\Phi\tilde{d}}\Big], \tag{3.4}$$

where N_c is the color factor ($N_c = 3(1)$ for final state quarks (leptons)) and the dΓ's correspond, respectively, to the separate contributions of the square of the gauge boson, $\tilde{u}$, $\tilde{d}$ and Higgs exchanges and the $V\tilde{u}$, $V\tilde{d}$, $\tilde{u}\tilde{d}$, $\Phi\tilde{u}$, $\Phi\tilde{d}$ and H_1H_2 interferences.

The various contributions, in terms of the couplings given in Sect. 2.2, read

$$\mathrm{d}\Gamma_V = \frac{4}{(y_3 - \mu_V)^2} \\ \times \Big\{[(v_V^f - a_V^f)^2 (G_{jiV}^{\mathrm{L}})^2 + (v_V^f + a_V^f)^2 \\ \times (G_{jiV}^{\mathrm{R}})^2] x_1 y_1 + [(v_V^f - a_V^f)^2 (G_{jiV}^{\mathrm{R}})^2 \\ + (v_V^f + a_V^f)^2 (G_{jiV}^{\mathrm{L}})^2] x_2 y_2 \\ - 4[(v_V^f)^2 + (a_V^f)^2] G_{jiV}^{\mathrm{L}} G_{jiV}^{\mathrm{R}} \sqrt{\mu_\chi} y_3\Big\}, \tag{3.5}$$

$$\mathrm{d}\Gamma_{\tilde{d}} = \sum_{k,l=1}^{2} \frac{x_1 y_1}{(1 - x_1 - \mu_{\tilde{d}_k})(1 - x_1 - \mu_{\tilde{d}_l})} \\ \times (a_{ik}^d a_{il}^d + b_{ik}^d b_{il}^d)(a_{jk}^d a_{jl}^d + b_{jk}^d b_{jl}^d), \tag{3.6}$$

$$\mathrm{d}\Gamma_{\tilde{u}} = \sum_{k,l=1}^{2} \frac{x_2 y_2}{(1 - x_2 - \mu_{\tilde{u}_k})(1 - x_2 - \mu_{\tilde{u}_l})} \\ \times (a_{ik}^u a_{il}^u + b_{ik}^u b_{il}^u)(a_{jk}^u a_{jl}^u + b_{jk}^u b_{jl}^u), \tag{3.7}$$

$$\mathrm{d}\Gamma_\Phi = 2\sum_k \frac{y_3\left[(v_k^f)^2 + (a_k^f)^2\right]}{(y_3 - \mu_{H_k})^2} \\ \times [((G_{ijk}^{\mathrm{L}})^2 + (G_{ijk}^{\mathrm{R}})^2) x_3 + 4\sqrt{\mu_\chi}(G_{ijk}^{\mathrm{L}} G_{ijk}^{\mathrm{R}})], \tag{3.8}$$

$$\mathrm{d}\Gamma_{H_1H_2} = \frac{4 y_3 v_1^f v_2^f}{(y_3 - \mu_{H_1})(y_3 - \mu_{H_2})} \Big[(G_{ij1}^{\mathrm{L}} G_{ij2}^{\mathrm{L}} + G_{ij1}^{\mathrm{R}} G_{ij2}^{\mathrm{R}}) x_3 \\ + 2\sqrt{\mu_\chi}(G_{ij1}^{\mathrm{L}} G_{ij2}^{\mathrm{R}} + G_{ij2}^{\mathrm{L}} G_{ij1}^{\mathrm{R}})\Big], \tag{3.9}$$

$$\mathrm{d}\Gamma_{V\tilde{d}} = -4\sum_{k=1}^{2} \Bigg\{\Big\{\Big(\,\big[a_{ik}^d a_{jk}^d G_{jiV}^{\mathrm{R}}(v_V^d + a_V^d) \\ + b_{ik}^d b_{jk}^d G_{jiV}^{\mathrm{L}}(v_V^d - a_V^d)\big]\, x_1 y_1\Big) \\ \Big/\Big((y_3 - \mu_V)(1 - x_1 - \mu_{\tilde{d}_k})\Big)\Big\} \\ - \Big\{\Big(\sqrt{\mu_\chi}\,\big[a_{ik}^d a_{jk}^d G_{jiV}^{\mathrm{L}}(v_V^d + a_V^d) \\ + b_{ik}^d b_{jk}^d G_{jiV}^{\mathrm{R}}(v_V^d - a_V^d)\big]\, y_3\Big) \\ \Big/\Big((y_3 - \mu_V)(1 - x_1 - \mu_{\tilde{d}_k})\Big)\Big\}\Bigg\} \tag{3.10}$$

$$\mathrm{d}\Gamma_{V\tilde{u}} = 4\sum_{k=1}^{2} \Bigg\{\Big\{\Big(\,\big[a_{ik}^u a_{jk}^u G_{jiV}^{\mathrm{L}}(v_V^u + a_V^u) \\ + b_{ik}^u b_{jk}^u G_{jiV}^{\mathrm{R}}(v_V^u - a_V^u)\big]\, x_2 y_2\Big) \\ \Big/\Big((y_3 - \mu_V)(1 - x_2 - \mu_{\tilde{u}_k})\Big)\Big\} \\ - \Big\{\Big(\sqrt{\mu_\chi}\,\big[a_{ik}^u a_{jk}^u G_{jiV}^{\mathrm{R}}(v_V^u + a_V^u) \\ + b_{ik}^u b_{jk}^u G_{jiV}^{\mathrm{L}}(v_V^u - a_V^u)\big]\, y_3\Big) \\ \Big/\Big((y_3 - \mu_V)(1 - x_2 - \mu_{\tilde{u}_k})\Big)\Big\}\Bigg\}, \tag{3.11}$$

$$\mathrm{d}\Gamma_{\tilde{u}\tilde{d}} = \sum_{k,l=1}^{2} \Bigg\{\Big\{\Big((a_{jk}^u a_{il}^d b_{ik}^u b_{jl}^d + a_{ik}^u a_{jl}^d b_{jk}^u b_{il}^d) \\ \times(-x_1 y_1 - x_2 y_2 + x_3 y_3)\Big) \\ \Big/\Big((1 - x_2 - \mu_{\tilde{u}_k})(1 - x_1 - \mu_{\tilde{d}_l})\Big)\Big\} \\ + \Big\{\Big(2(a_{ik}^u a_{jk}^u a_{il}^d a_{jl}^d + b_{ik}^u b_{jk}^u b_{il}^d b_{jl}^d)\sqrt{\mu_\chi} y_3\Big)$$

leptons, since χ_2^0 and χ_1^+ are expected to have masses larger than $\mathcal{O}(100\,\mathrm{GeV})$. The approximation would be bad for top quark final states; however, if the three-body decays $\chi_2^0 \to \chi_1^0 t\bar{t}$ and $\chi_1^+ \to \chi_1^0 t\bar{b}$ are kinematically allowed, they will not play a major role since the charginos and neutralinos will have enough phase space to decay first into the two-body channels $\chi_2^0 \to \chi_1^0 Z$, $\chi_1^0 h$ (and possibly $\chi_1^0 H$ and $\chi_1^0 A$) and $\chi_1^+ \to \chi_1^0 W$ (and possibly $\chi_1^0 H^+$), which will be largely dominating

$$\Big/\Big((1-x_2-\mu_{\tilde{u}_k})(1-x_1-\mu_{\tilde{d}_l})\Big)\Big\}\Bigg\}, \quad (3.12)$$

$$\begin{aligned}\mathrm{d}\Gamma_{\Phi\tilde{d}} = -\sum_{k,l}\Bigg\{&\Big\{\Big((v_k^d-a_k^d)a_{il}^d b_{jl}^d\,\big(G_{ijk}^{\mathrm{R}}\\ &\times(x_1y_1-x_2y_2+x_3y_3)+2G_{ijk}^{\mathrm{L}}\sqrt{\mu_\chi}y_3\big)\Big)\\ &\Big/\Big((y_3-\mu_k)(1-x_1-\mu_{\tilde{d}_l})\Big)\Big\}\\ &+\Big\{\Big((a_k^d+v_k^d)b_{il}^d a_{jl}^d\,\big(G_{ijk}^{\mathrm{L}}(x_1y_1-x_2y_2+x_3y_3)\\ &+2G_{ijk}^{\mathrm{R}}\sqrt{\mu_\chi}y_3\big)\Big)\\ &\Big/\Big((y_3-\mu_k)(1-x_1-\mu_{\tilde{d}_l})\Big)\Big\}\Bigg\}, \end{aligned} \quad (3.13)$$

$$\begin{aligned}\mathrm{d}\Gamma_{\Phi\tilde{u}} = \sum_{k,l}\Bigg\{&\Big\{\Big((v_k^u-a_k^u)b_{il}^u a_{jl}^u\,\big(G_{ijk}^{\mathrm{R}}\\ &\times(x_1y_1-x_2y_2-x_3y_3)-2G_{ijk}^{\mathrm{L}}\sqrt{\mu_\chi}y_3\big)\Big)\\ &\Big/\Big((y_3-\mu_k)(1-x_2-\mu_{\tilde{u}_l})\Big)\Big\}\\ &+\Big\{\Big((a_k^u+v_k^u)a_{il}^u b_{jl}^u\,\big(G_{ijk}^{\mathrm{L}}(x_1y_1-x_2y_2-x_3y_3)\\ &-2G_{ijk}^{\mathrm{R}}\sqrt{\mu_\chi}y_3\big)\Big)\\ &\Big/\Big((y_3-\mu_k)(1-x_2-\mu_{\tilde{u}_l})\Big)\Big\}\Bigg\}. \end{aligned} \quad (3.14)$$

A few remarks need to be made at this stage.
(1) In the expressions of the couplings, the indices i and j refer always to the decaying chargino or neutralino and the final state neutralino, respectively.
(2) For the Higgs boson exchange contributions, in the case of chargino decays, only the exchange of the charged Higgs boson is present and in $\mathrm{d}\Gamma_\Phi$ one has $k=4$ only. In the case of neutralino decays, the three neutral Higgs bosons will contribute and k in the sum $\sum_k$ of $\mathrm{d}\Gamma_\Phi$ runs from $k=1$ to 3. In addition, there is an extra term, $\mathrm{d}\Gamma_{H_1H_2}$, due to the interference between the exchange of the two CP-even Higgs bosons h and H. Note also that in this case, there is a difference between the contributions of the CP-even (and the charged) and CP-odd Higgs bosons which appears in the terms $\epsilon_{1,2,4}=1$ and $\epsilon_3=-1$ in the couplings.
(3) For massless final state fermions, there is no interference between the vector boson and Higgs boson contributions. In the appendix, where the fermion mass dependence will be included, interference terms between the Higgs bosons and the vector bosons, which are proportional to the fermion masses, will be shown explicitly.
(4) In the sfermion exchange diagrams, there is a relative minus sign between the amplitudes of the u- and t-channels, due to Wick's theorem. This leads to $\mathrm{d}\Gamma_{V\tilde{u}}$ and $\mathrm{d}\Gamma_{V\tilde{d}}$ contributions which are anti-symmetric in the interchange of x_1 and x_2. In the case of $\mathrm{d}\Gamma_{\Phi\tilde{u}}$ and $\mathrm{d}\Gamma_{\Phi\tilde{d}}$, the contributions are symmetric in the interchange of x_1 and x_2, due to the scalar nature of the Higgs bosons.

3.2 Integrated three-body partial widths

Integrating over the energies x_1 and x_2 of the two fermions, with boundary conditions,

$$1-x_1-\mu_\chi \le x_2 \le 1-\frac{\mu_\chi}{1-x_1}, \qquad 0\le x_1\le 1-\mu_\chi, \quad (3.15)$$

one obtains the partial decay width, which is given by an expression similar to (3.4):

$$\begin{aligned}\Gamma_{\chi_i} = \frac{\alpha^2 N_c}{32\pi}m_{\chi_i}\Big[&\Gamma_V+\Gamma_{\tilde{u}}+\Gamma_{\tilde{d}}+\Gamma_\Phi+\Gamma_{H_1H_2}+\Gamma_{V\tilde{u}}\\ &+\Gamma_{V\tilde{d}}+\Gamma_{\tilde{u}\tilde{d}}+\Gamma_{\Phi\tilde{u}}+\Gamma_{\Phi\tilde{d}}\Big].\end{aligned} \quad (3.16)$$

Using the phase space functions λ_k and the function $\mathcal{L}_k$ defined by

$$\lambda_k = 1-2\mu_\chi-2\mu_k+(\mu_k-\mu_\chi)^2, \quad (3.17)$$

$$\begin{aligned}\mathcal{L}_k = \frac{2}{\sqrt{-\lambda_k}}\Bigg[&\mathrm{Arctan}\left(\frac{-1+\mu_\chi-\mu_k}{\sqrt{-\lambda_k}}\right)\\ &-\mathrm{Arctan}\left(\frac{1-\mu_\chi-\mu_k}{\sqrt{-\lambda_k}}\right)\Bigg],\end{aligned} \quad (3.18)$$

one has for the various contributions:

$$\begin{aligned}\Gamma_V = 8&\left[(v_V^f)^2+(a_V^f)^2\right]\left[(G_{jiV}^{\mathrm{L}})^2+(G_{jiV}^{\mathrm{R}})^2\right]\\ &\times\Bigg\{\frac{\mu_\chi-1}{6\mu_V}(\lambda_V+\mu_V(5+5\mu_\chi-7\mu_V))\\ &-\frac{\mu_V}{2}(1+\mu_\chi-\mu_V)\mathrm{Log}\mu_\chi-\frac{\mu_V}{2}(\lambda_V+2\mu_\chi)\mathcal{L}_V\Bigg\}\\ &-8\left[(v_V^f)^2+(a_V^f)^2\right]G_{jiV}^{\mathrm{L}}G_{jiV}^{\mathrm{R}}\sqrt{\mu_\chi}\\ &\times\Big\{4(\mu_\chi-1)+(1+\mu_\chi-2\mu_V)\mathrm{Log}\mu_\chi\\ &+(\lambda_V-\mu_V(1+\mu_\chi-\mu_V))\mathcal{L}_V\Big\},\end{aligned} \quad (3.19)$$

$$\begin{aligned}\Gamma_{\tilde{f}} = \sum_{k,l=1}^{2}&(a_{ik}^f a_{il}^f+b_{ik}^f b_{il}^f)(a_{jk}^f a_{jl}^f+b_{jk}^f b_{jl}^f)\\ &\times\Bigg\{(1-\mu_\chi)(\mu_{\tilde{f}_k}+\mu_{\tilde{f}_l})-\frac{3}{2}(1-\mu_\chi^2)\\ &+\frac{(\mu_{\tilde{f}_l}-1)^2(\mu_{\tilde{f}_l}-\mu_\chi)^2}{\mu_{\tilde{f}_l}(\mu_{\tilde{f}_l}-\mu_{\tilde{f}_k})}\mathrm{Log}\frac{\mu_{\tilde{f}_l}-1}{\mu_{\tilde{f}_l}-\mu_\chi}\\ &+\frac{(\mu_{\tilde{f}_k}-1)^2(\mu_{\tilde{f}_k}-\mu_\chi)^2}{\mu_{\tilde{f}_k}(\mu_{\tilde{f}_k}-\mu_{\tilde{f}_l})}\mathrm{Log}\frac{\mu_{\tilde{f}_k}-1}{\mu_{\tilde{f}_k}-\mu_\chi}\\ &-\frac{\mu_\chi^2}{\mu_{\tilde{f}_k}\mu_{\tilde{f}_l}}\mathrm{Log}\mu_\chi\Bigg\},\end{aligned} \quad (3.20)$$

$$\Gamma_\Phi = \sum_k 2\left[(v_k^f)^2+(a_k^f)^2\right]\left[(G_{ijk}^{\mathrm{L}})^2+(G_{ijk}^{\mathrm{R}})^2\right]$$

$$
\begin{aligned}
&\times \Big\{ \frac{1}{2}(1-\mu_\chi)(6\mu_k - 5 - 5\mu_\chi) \\
&+ \frac{1}{2}\left[-5\mu_\chi^2\mu_k - 3\mu_k^3 + 7\mu_k^2 + 1 - \mu_\chi^2 - \mu_\chi + \mu_\chi^3 \right.\\
&\left. - 5\mu_k + 7\mu_\chi\mu_k^2 - 2\mu_\chi\mu_k\right]\mathcal{L}_k \\
&+ \frac{1}{2}\left(1 - 4\mu_k - 4\mu_\chi\mu_k + 3\mu_k^2 + \mu_\chi^2\right)\mathrm{Log}\mu_\chi \Big\} \\
&+ 4\left[(v_k^f)^2 + (a_k^f)^2\right] G^{\mathrm{L}}_{ijk} G^{\mathrm{R}}_{ijk}\sqrt{\mu_\chi} \\
&\times \{4(\mu_\chi - 1) + (1 + \mu_\chi - 2\mu_k)\mathrm{Log}\mu_\chi \\
&+ (\lambda_k - \mu_k(1 + \mu_\chi - \mu_k))\mathcal{L}_k\},
\end{aligned} \tag{3.21}
$$

$$
\begin{aligned}
\Gamma_{H_1H_2} &= 2v_1^f v_2^f \Big\{ (G^{\mathrm{L}}_{ij1}G^{\mathrm{L}}_{ij2} + G^{\mathrm{R}}_{ij1}G^{\mathrm{R}}_{ij2}) \\
&\times \Big[(2\mu_{H_1} + 2\mu_{H_2} - 3\mu_\chi - 3)(1 - \mu_\chi) \\
&+ \frac{\mu_{H_1}(1 + \mu_\chi - \mu_{H_1})}{\mu_{H_2} - \mu_{H_1}}\lambda_{H_1}\mathcal{L}_{H_1} \\
&- \frac{\mu_{H_2}(1 + \mu_\chi - \mu_{H_2})}{\mu_{H_2} - \mu_{H_1}}\lambda_{H_2}\mathcal{L}_{H_2} \\
&+ (1 + \mu_\chi^2 + \mu_{H_1}^2 + \mu_{H_2}^2 + \mu_{H_1}\mu_{H_2} \\
&- 2(1 + \mu_\chi)(\mu_{H_1} + \mu_{H_2}))\mathrm{Log}\mu_\chi \Big] \\
&+ 2\sqrt{\mu_\chi}(G^{\mathrm{L}}_{ij1}G^{\mathrm{R}}_{ij2} + G^{\mathrm{L}}_{ij2}G^{\mathrm{R}}_{ij1}) \\
&\times \Big[\frac{\mu_{H_1}}{\mu_{H_2} - \mu_{H_1}}\lambda_{H_1}\mathcal{L}_{H_1} - \frac{\mu_{H_2}}{\mu_{H_2} - \mu_{H_1}}\lambda_{H_2}\mathcal{L}_{H_2} \\
&+ \frac{\mu_{H_1}^2 - \mu_{H_2}^2 - (\mu_{H_1} - \mu_{H_2})(1 + \mu_\chi)}{\mu_{H_2} - \mu_{H_1}}\mathrm{Log}\mu_\chi \\
&- 2(1 - \mu_\chi) \Big] \Big\}
\end{aligned} \tag{3.22}
$$

$$
\begin{aligned}
\Gamma_{V\tilde{f}} &= 4\sum_{k=1}^{2} \Big\{ A_1^f \Big(\frac{\mu_\chi - 1}{4}(\mu_\chi + 1 - 4\mu_{\tilde{f}_k} + 2\mu_V) \\
&+ \frac{1}{4}(1 + \mu_\chi - 2\mu_{\tilde{f}_k} + \mu_V)\lambda_V\mathcal{L}_V \\
&+ \frac{1}{4}(1 + 4\mu_\chi + \mu_\chi^2 - 2\mu_{\tilde{f}_k} - 2\mu_{\tilde{f}_k}\mu_\chi \\
&+ 2\mu_{\tilde{f}_k}\mu_V - \mu_V^2)\mathrm{Log}\mu_\chi \\
&+ (\mu_\chi - \mu_{\tilde{f}_k})(-1 + \mu_{\tilde{f}_k})\mathcal{F}(a_+^V, a_-^V, \mu_{\tilde{f}_k}, \mu_V) \Big) \\
&- A_2^f\sqrt{\mu_\chi}\Big(\mu_\chi - 1 - \frac{\mu_\chi}{\mu_{\tilde{f}_k}}\mathrm{Log}\mu_\chi \\
&+ \mu_V\mathcal{F}(a_+^V, a_-^V, \mu_{\tilde{f}_k}, \mu_V) - \frac{1}{\mu_{\tilde{f}_k}}(\mu_\chi - \mu_{\tilde{f}_k} \\
&- \mu_\chi\mu_{\tilde{f}_k} + \mu_{\tilde{f}_k}^2)\mathrm{Log}\frac{\mu_{\tilde{f}_k} - 1}{\mu_{\tilde{f}_k} - \mu_\chi} \Big) \Big\},
\end{aligned} \tag{3.23}
$$

where

$$
\begin{aligned}
A_1^d &= -[a_{ik}^d a_{jk}^d G^{\mathrm{R}}_{jiV}(v_V^d + a_V^d) + b_{ik}^d b_{jk}^d G^{\mathrm{L}}_{jiV}(v_V^d - a_V^d)], \\
A_1^u &= a_{ik}^u a_{jk}^u G^{\mathrm{L}}_{jiV}(v_V^u + a_V^u) + b_{ik}^u b_{jk}^u G^{\mathrm{R}}_{jiV}(v_V^u - a_V^u), \\
A_2^d &= -[a_{ik}^d a_{jk}^d G^{\mathrm{L}}_{jiV}(v_V^d + a_V^d) + b_{ik}^d b_{jk}^d G^{\mathrm{R}}_{jiV}(v_V^d - a_V^d)], \\
A_2^u &= a_{ik}^u a_{jk}^u G^{\mathrm{R}}_{jiV}(v_V^u + a_V^u) \\
&\quad + b_{ik}^u b_{jk}^u G^{\mathrm{L}}_{jiV}(v_V^u - a_V^u),
\end{aligned} \tag{3.24}
$$

$$
\begin{aligned}
\Gamma_{\Phi\tilde{f}} &= \sum_{k,l}\Big\{ B_1^f \Big((1 - \mu_\chi)(-1 + 2\mu_k - \mu_\chi + 2\mu_{\tilde{f}_l}) \\
&- \mu_k\lambda_k\mathcal{L}_k - \mu_k(1 + \mu_\chi - \mu_k)\mathrm{Log}\mu_\chi \\
&- 2\mathrm{Log}\frac{\mu_{\tilde{f}_l} - \mu_\chi}{\mu_{\tilde{f}_l} - 1}(\mu_\chi - \mu_{\tilde{f}_l} - \mu_\chi\mu_{\tilde{f}_l} + \mu_{\tilde{f}_l}^2) \\
&- 2\mu_k\mu_{\tilde{f}_l}\mathcal{F}(a_+^{H_k}, a_-^{H_k}, \mu_{\tilde{f}_l}, \mu_k) \Big) \\
&- 2B_2^f\sqrt{\mu_\chi}\Big(\mu_\chi - 1 - \frac{\mu_\chi}{\mu_{\tilde{f}_l}}\mathrm{Log}\mu_\chi \\
&- \frac{1}{\mu_{\tilde{f}_l}}(\mu_\chi - \mu_{\tilde{f}_l} - \mu_\chi\mu_{\tilde{f}_l} + \mu_{\tilde{f}_l}^2)\mathrm{Log}\frac{\mu_{\tilde{f}_l} - 1}{\mu_{\tilde{f}_l} - \mu_\chi} \\
&+ \mu_k\mathcal{F}(a_+^{H_k}, a_-^{H_k}, \mu_{\tilde{f}_l}, \mu_k) \Big) \Big\},
\end{aligned} \tag{3.25}
$$

where

$$
\begin{aligned}
B_1^d &= (v_k^d - a_k^d)a_{il}^d b_{jl}^d G^{\mathrm{R}}_{ijk} + (v_k^d + a_k^d)b_{il}^d a_{jl}^d G^{\mathrm{L}}_{ijk}, \\
B_1^u &= (v_k^u - a_k^u)a_{jl}^u b_{il}^u G^{\mathrm{R}}_{ijk} + (v_k^u + a_k^u)a_{il}^u b_{jl}^u G^{\mathrm{L}}_{ijk}, \\
B_2^d &= (v_k^d - a_k^d)a_{il}^d b_{jl}^d G^{\mathrm{L}}_{ijk} + (v_k^d + a_k^d)b_{il}^d a_{jl}^d G^{\mathrm{R}}_{ijk}, \\
B_u^2 &= (v_k^u - a_k^u)b_{il}^u a_{jl}^u G^{\mathrm{L}}_{ijk} \\
&\quad + (v_k^u + a_k^u)a_{il}^u b_{jl}^u G^{\mathrm{R}}_{ijk},
\end{aligned} \tag{3.26}
$$

$$
\begin{aligned}
\Gamma_{\tilde{u}\tilde{d}} &= -2\sum_{k,l=1}^{2}\Big\{ (a_{jk}^u a_{il}^d b_{ik}^u b_{jl}^d + a_{ik}^u a_{jl}^d b_{jk}^u b_{il}^d) \\
&\times \Big(\frac{1}{2}(\mu_\chi - 1)(2\mu_{\tilde{u}_k} + 2\mu_{\tilde{d}_l} - \mu_\chi - 1) - \mu_\chi\mathrm{Log}\mu_\chi \\
&+ \mathrm{Log}\frac{\mu_{\tilde{u}_k} - 1}{\mu_{\tilde{u}_k} - \mu_\chi}(\mu_{\tilde{u}_k} - \mu_\chi)(1 - \mu_{\tilde{u}_k}) \\
&+ \mathrm{Log}\frac{\mu_{\tilde{d}_l} - 1}{\mu_{\tilde{d}_l} - \mu_\chi}(\mu_{\tilde{d}_l} - \mu_\chi)(1 - \mu_{\tilde{d}_l}) \\
&+ (\mu_\chi - \mu_{\tilde{u}_k}\mu_{\tilde{d}_l}) \\
&\times \tilde{\mathcal{F}}((\mu_{\tilde{u}_k} - \mu_\chi)/\mu_{\tilde{u}_k}, \mu_{\tilde{u}_k} - \mu_\chi, \mu_{\tilde{d}_l}, \mu_{\tilde{u}_k}) \Big) \\
&+ (a_{ik}^u a_{jk}^u a_{il}^d a_{jl}^d + b_{ik}^u b_{jk}^u b_{il}^d b_{jl}^d) \\
&\times \sqrt{\mu_\chi}\Big(\mathrm{Log}\frac{\mu_{\tilde{u}_k} - 1}{\mu_{\tilde{u}_k} - \mu_\chi}(\mu_{\tilde{u}_k} - \mu_\chi)(1 - \mu_{\tilde{u}_k})/\mu_{\tilde{u}_k} \\
&+ \mathrm{Log}\frac{\mu_{\tilde{d}_l} - 1}{\mu_{\tilde{d}_l} - \mu_\chi}(\mu_{\tilde{d}_l} - \mu_\chi)(1 - \mu_{\tilde{d}_l})/\mu_{\tilde{d}_l}
\end{aligned}
$$

$$+ 2(\mu_\chi - 1) + (1 + \mu_\chi - \mu_{\tilde{d}_l} - \mu_{\tilde{u}_k})$$
$$\times \tilde{\mathcal{F}}((\mu_{\tilde{u}_k} - \mu_\chi)/\mu_{\tilde{u}_k}, \mu_{\tilde{u}_k} - \mu_\chi, \mu_{\tilde{d}_l}, \mu_{\tilde{u}_k})$$
$$- \left(\frac{\mu_\chi}{\mu_{\tilde{u}_k}} + \frac{\mu_\chi}{\mu_{\tilde{d}_l}}\right) \mathrm{Log}\mu_\chi \Bigg) \Bigg\}. \quad (3.27)$$

In the previous expressions, we have used the variables and functions:

$$a^i_\pm = \frac{1}{2}(1 - \mu_\chi + \mu_i \pm \sqrt{\lambda_i}), \quad (3.28)$$
$$\mathcal{F}(a, b, \mu_i, \mu_j) = f(a, \mu_i) + f(b, \mu_i) - f(1, \mu_i) + \mathrm{Log}\mu_j \mathrm{Log}\frac{\mu_i - \mu_\chi}{\mu_i - 1},$$
$$\tilde{\mathcal{F}}(a, b, \mu_i, \mu_j) = f(a, \mu_i) - f(b, \mu_i) - f(1, \mu_i) - \mathrm{Log}\mu_j \mathrm{Log}\frac{\mu_i - \mu_\chi}{\mu_i - 1}, \quad (3.29)$$
$$f(a, \mu_i) = \mathrm{Li}_2\left(\frac{\mu_i - \mu_\chi}{a + \mu_i - 1}\right) - \mathrm{Li}_2\left(\frac{\mu_i - 1}{a + \mu_i - 1}\right) - \mathrm{Log}(a + \mu_i - 1)\mathrm{Log}\left(\frac{\mu_i - \mu_\chi}{\mu_i - 1}\right), \quad (3.30)$$

where Li_2 is the Spence function defined by $\mathrm{Li}_2(x) = \int_0^1 t^{-1}\mathrm{Log}(1 - xt)\mathrm{d}t$.

3.3 The two-body partial decay widths

The two-body partial decay widths can be obtained from the expressions given in Sect. 3.1 by including the total decay widths of the exchanged gauge and Higgs bosons and the sfermions. In this case a smooth transition between three- and two-body partial decay widths can be obtained. We will list below the integrated form of the two-body partial decay widths of charginos and neutralinos into sfermion–fermion pairs (with massive fermions), and into neutralino and gauge or Higgs boson final states; see also [33].

$$\Gamma(\chi_i \to f\tilde{f}_j) = \frac{\alpha N_c}{8} m_{\chi_i} \left[\left((a^f_{ij})^2 + (b^f_{ij})^2 \right) \times (1 - \mu_{\tilde{f}_j} + \mu_f) + 4\sqrt{\mu_f} a^f_{ij} b^f_{ij} \right] \times \lambda^{1/2}(\mu_f, \mu_{\tilde{f}_j}), \quad (3.31)$$

$$\Gamma(\chi_i \to \chi_j V) = \frac{\alpha}{8} m_{\chi_i} \lambda^{1/2}(\mu_{\chi_j}, \mu_V) \Big\{ -12\sqrt{\mu_{\chi_j}} G^{\mathrm{L}}_{jiV} \times G^{\mathrm{R}}_{jiV} + \left[(G^{\mathrm{L}}_{jiV})^2 + (G^{\mathrm{R}}_{jiV})^2 \right] \times (1 + \mu_{\chi_j} - \mu_V) + (1 - \mu_{\chi_j} + \mu_V) \times (1 - \mu_{\chi_j} - \mu_V)\mu_V^{-1} \Big\}, \quad (3.32)$$

$$\Gamma(\chi_i \to \chi_j H_k) = \frac{\alpha}{8} m_{\chi_i} \lambda^{1/2}(\mu_{\chi_j}, \mu_{H_k}) \times \Big\{ \left[(G^{\mathrm{L}}_{ijk})^2 + (G^{\mathrm{R}}_{ijk})^2 \right] (1 + \mu_{\chi_j} - \mu_{H_k}) + 4\sqrt{\mu_{\chi_j}} G^{\mathrm{L}}_{ijk} G^{\mathrm{R}}_{ijk} \Big\}, \quad (3.33)$$

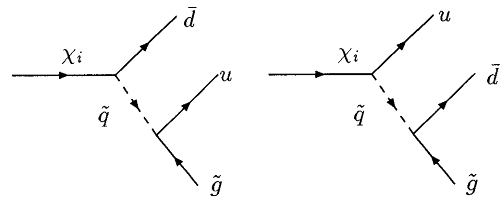

Fig. 5. The Feynman diagrams contributing to the three-body decay $\chi_i \to \tilde{g}q\bar{q}$

with

$$\lambda(x, y) = 1 + x^2 + y^2 - 2x - 2y - 2xy,$$
$$\mu_X = m^2_X / m^2_{\chi_i}, \quad (3.34)$$

3.4 Decays into gluino and quark–antiquark final states

As discussed in Sect. 2.1, in models without gaugino mass unification at the GUT scale, the lightest chargino and the next-to-lightest neutralino could be heavier than the gluino. In this case, the three-body decay modes

$$\chi_i \to \tilde{g}u\bar{d}, \quad (3.35)$$

with $\chi_i \equiv \chi_1^\pm$ or χ_2^0, are kinematically accessible. This decay is mediated by t- and u-channel exchange of squarks only; see Fig. 5.

The Dalitz density and the partial decay width, neglecting the masses of the final state quarks, are given by

$$\frac{\mathrm{d}\Gamma_{\chi_i}}{\mathrm{d}x_1 \mathrm{d}x_2} = \frac{e^2 g_s^2 m_{\chi_i}}{8(2\pi)^3} [\mathrm{d}\Gamma_{\tilde{u}} + \mathrm{d}\Gamma_{\tilde{d}} + \mathrm{d}\Gamma_{\tilde{u}\tilde{d}}],$$
$$\Gamma_{\chi_i} = \frac{\alpha\alpha_s}{4\pi} m_{\chi_i} [\Gamma_{\tilde{u}} + \Gamma_{\tilde{d}} + \Gamma_{\tilde{u}\tilde{d}}], \quad (3.36)$$

with $x_1 = 2E_u/m_{\chi_i}, x_2 = 2E_d/m_{\chi_i}$. The various amplitudes are as in (3.6), (3.7) and (3.12) for the Dalitz densities and (3.20) and (3.27) for the integrated width, now with $\mu_\chi \equiv m^2_{\tilde{g}}/m^2_{\chi_i}$. One has also to replace the final neutralino–f–$\tilde{f}_l$ couplings, a^f_{jl}, b^f_{jl}, by the gluino–quark–squark couplings, a^q_l, b^q_l, which in the case of mixing read

$$a^q_1 = b^q_2 = \sin\theta_q, \quad a^q_2 = -b^q_1 = \cos\theta_q. \quad (3.37)$$

The expressions for the three-body decays of gluinos into the $\chi_i + q\bar{q}$ final states [31,9] are given by the previous formulae with the interchange of m_{χ_i} and $m_{\tilde{g}}$ and by dividing the result by a factor of 8 to account for the color numbers of the gluino. (Note that this factor is missing in the expression of the gluino decay width in [9]; since it is a global factor, the branching ratios are therefore not affected.)

4 Numerical analysis

We will first illustrate our results in an mSUGRA-type model, where we assume a universal mass m_0 for the scalar fermions and a mass $m_{1/2}$ for the gauginos at the GUT scale; the soft SUSY-breaking masses for the Higgs bosons are however disconnected from the one of the sfermions so that the pseudo-scalar Higgs boson mass M_A and the higgsino parameter μ are free parameters (in contrast to the mSUGRA model where μ is determined, up to its sign, from the requirement of electroweak symmetry breaking). For the squark sector, we will use the simple expressions (2.30) for the soft SUSY-breaking left- and right-handed squark and slepton masses when performing the RGE evolution to the weak scale at one-loop order if the Yukawa couplings in the RGE's are neglected[5]. One has then, in the case of the third generation sparticles, to include the mixing. Since for sbottoms and stau's, large enough off-diagonal elements of the mass matrices are obtained only for large μ and $\tan\beta$ values and the trilinear couplings play only a marginal role, we will fix the latter at $A_b = A_\tau = -500$ GeV in the entire analysis. We will choose two representative values for $\tan\beta$: a "low" value ($\tan\beta = 5$) and a large value ($\tan\beta = 50$) and two values for the pseudoscalar A boson mass[6], $M_A = 100$ and 500 GeV.

In a second step, we will relax the gaugino mass unification constraint $m_1 = m_2 = m_3 = m_{1/2}$ at the GUT scale, and use the weak scale gaugino masses M_1 and M_2 given in Table 1 for the F_Φ representations and the **OII** model. We will still use the soft SUSY-breaking scalar masses given in (2.30). In this case, we will stick in the illustrations to the large $\tan\beta$ scenario, $\tan\beta = 50$, but still show the effect of the Higgs boson contribution by taking the two examples $M_A = 100$ and 500 GeV.

In most of the cases, the wino mass parameter will be fixed to $M_2 = 150$ GeV, which for large values of μ, leads in an mSUGRA-type model to the masses $m_{\chi_1^\pm} \simeq m_{\chi_2^0} \simeq$ 150 GeV and $m_{\chi_1^0} \simeq 75$ GeV (there is a very small variation with the value of $\tan\beta$) and hence to states which are accessible at the high-luminosity phase of the Tevatron and at a future e^+e^- linear collider with a c.m. energy of 500 GeV.

Note that in the entire analysis, we will include the radiative corrections to the b-quark and τ-lepton masses, as well as the radiative corrections to the chargino, neutralino and gluino masses given in Sect. 2.1. We will also take into account the full dependence on the final state fermion masses (using the pole masses $m_b = 4.6$ GeV, $m_\tau = 1.78$ GeV and $m_c = 1.45$ GeV in the phase space, the other fermions are taken to be massless) since in some cases (in particular when the decaying chargino or neutralino has a mass which is close to the final LSP mass), they play a significant role.

The branching ratios for the lightest chargino $\chi_1^\pm$ and next-to-lightest neutralino χ_2^0 into the LSP and τ and b-quark final states are shown in Figs. 6–8, in model **1** with gaugino mass unification at $M_{\rm GUT}$. The wino mass parameter is fixed at $M_2 = 150$ GeV and the choices $\tan\beta = 5, 50$ and $M_A = 100, 500$ GeV have been made.

In Fig. 6a, BR($\chi_1^+ \to \chi_1^0\tau^+\nu$) is shown as a function of the lightest $\tilde\tau_1$ mass for $\mu = +500$ GeV. For large values of $m_{\tilde\tau_1}$ and with a heavy charged Higgs boson ($M_A =$ 500 GeV leading to $M_{H^\pm} = 506$ GeV), the branching ratio is small, being at the level of 10%. In this regime, the dominant contribution is coming from the virtual W-boson exchange and BR(χ_1^+) is practically the same as BR($W \to f\bar f$), i.e. $\sim 10\%$ for the $\tau^+\nu$ final state. However, for large values of $\tan\beta$ and for a light $H^\pm$-boson ($M_A = 100$ GeV leading to $M_{H^\pm} \sim 128$ GeV), the charged Higgs boson contribution (since the $H^\pm\nu\tau^\mp$ couplings are enhanced) becomes dominant and the fraction BR($\chi_1^+ \to \chi_1^0\tau^+\nu$) can reach the level of 40% even for $m_{\tilde\tau_1} \sim 500$ GeV. For smaller values of $m_{\tilde\tau_1}$, the virtual stau exchange diagram becomes more and more dominant, and BR($\chi_1^+ \to \chi_1^0\tau^+\nu$) becomes close to $\sim 80\%$ for stau masses of the order of 150 GeV. If in addition, $H^\pm$ is relatively light, the branching ratio reaches the level of 100%.

Figures 6b,c, where BR($\chi_1^+ \to \chi_1^0\tau^+\nu$) is plotted for a common sfermion mass $m_0 = 300$ GeV as a function of μ and $\tan\beta$, respectively, show the same trend from a different perspective. For small values of $\tan\beta$, the mixing in the stau sector and the Yukawa couplings of the τ-lepton are not enhanced and the branching fraction is at the level of 10%. But for large $\tan\beta$ values, the stau becomes light and the branching ratio becomes close to unity for large values of μ. This occurs more quickly, if the charged Higgs boson is light.

Figures 7 and 8 show, respectively, the branching ratios BR($\chi_2^0 \to \chi_1^0\tau^+\tau^-$) and BR($\chi_2^0 \to \chi_1^0 b\bar b$), as functions of $m_{\tilde\tau_1}$ or $m_{\tilde b_1}$ for $\mu = 500$ GeV (a), as a function of μ (b) and as a function of $\tan\beta$ (c) for $m_0 = 300$ GeV. In this case, there is a competition between $b\bar b$ and $\tau^+\tau^-$ final states. In the case of a light A-boson and for large tb values, the A and h contributions are much more important in the decay $\chi_2^0 \to \chi_1^0 b\bar b$ than in the channel $\chi_2^0 \to \chi_1^0\tau^+\tau^-$ because of the larger b-quark mass and the color factor; the Higgs contribution makes then BR($\chi_2^0 \to \chi_1^0 b\bar b$) dominating, except when $\tilde\tau_1$ is very light, and the two-body decay $\chi_2^0 \to \tilde\tau_1\tau$ is close to occur, making BR($\chi_2^0 \to \chi_1^0\tau^+\tau^-$) close to unity. Even for heavy A, H-bosons, BR($\chi_2^0 \to \chi_1^0 b\bar b$) can reach the level of $\sim 50\%$. However, for large enough values of $\tan\beta$ and μ, it is the decay channel $\chi_2^0 \to \chi_1^0\tau^+\tau^-$ which dominates, since for a universal scalar mass m_0, the stau is always lighter than the $\tilde b_1$-state and its virtual contribution is larger, despite of the color factor. Needless to say, the sum of the two branching ratios, BR($\chi_2^0 \to \chi_1^0\tau^+\tau^- + \chi_1^0 b\bar b$) is in general close to unity.

[5] As mentioned previously, for third generation sfermions, neglecting the Yukawa couplings in the RGE is a poor approximation since these couplings can be large; this is particularly the case for top squarks which however will not be considered in the present analysis, since we will assume that charginos and neutralinos are not heavy enough to decay into top quark final states

[6] In the large $\tan\beta$ scenario and in the non-decoupling regime, the experimental bounds on the masses of the pseudoscalar Higgs boson A and the lightest Higgs boson h in the MSSM from negative searches at LEP2 are $M_A, M_h \gtrsim$ 93.5 GeV [11]

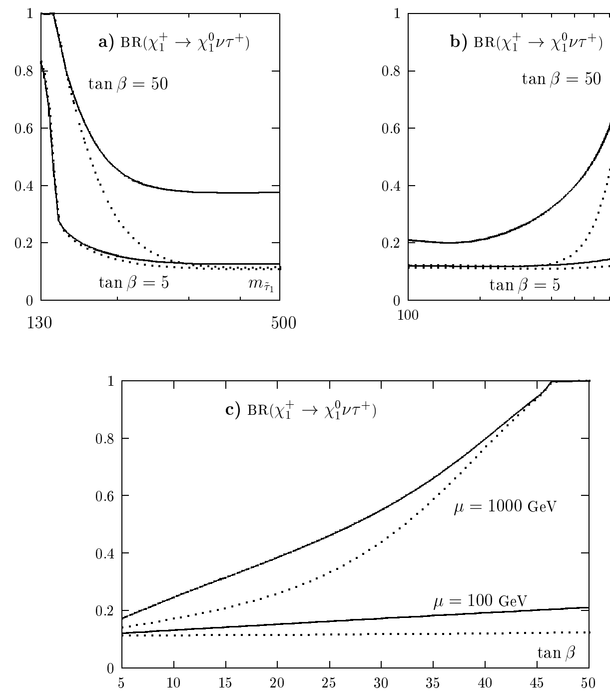

Fig. 6a–c. The branching ratio BR($\chi_1^+ \to \chi_1^0 \nu \tau^+$) for two values of $\tan\beta = 5$ and 50 and two values of $M_A = 100\,\mathrm{GeV}$ (solid lines) and 500 GeV (dashed lines) as a function of $m_{\tilde{\tau}_1}$ for $\mu = 500\,\mathrm{GeV}$ **a** as a function of μ assuming $m_0 = 300\,\mathrm{GeV}$ **b** and as a function of $\tan\beta$ for two values of $\mu = 100$ and 1000 GeV **c**; M_2 is fixed at 150 GeV

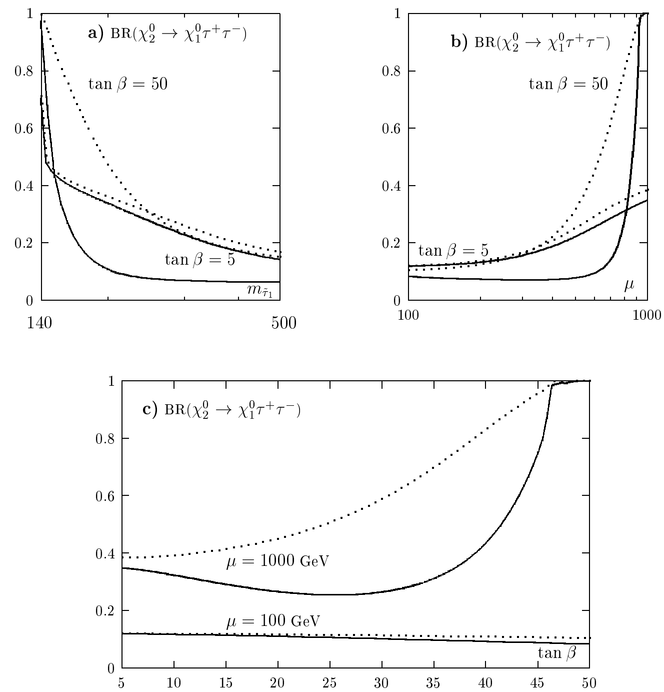

Fig. 7a–c. The branching ratio BR($\chi_2^0 \to \chi_1^0 \tau^+ \tau^-$) for two values of $\tan\beta = 5$ and 50 and two values of $M_A = 100\,\mathrm{GeV}$ (dashed lines) and 500 GeV (solid lines) as a function of $m_{\tilde{\tau}_1}$ for $\mu = 500\,\mathrm{GeV}$ **a** as a function of μ assuming $m_0 = 300\,\mathrm{GeV}$ **b** and as a function of $\tan\beta$ for two values of $\mu = 100$ and 1000 GeV **c**; M_2 is fixed at 150 GeV

In Fig. 9, we illustrate the effect of the radiative corrections to the b-quark mass (and to a lesser extent the tau-lepton mass) by showing the branching ratios BR($\chi_2^0 \to \chi_1^0 \tau^+ \tau^-$) and BR($\chi_2^0 \to \chi_1^0 b\bar{b}$) as a function of $\tan\beta$ with μ, m_0, M_A and M_2 fixed at, respectively, the values 1 TeV, 300 GeV, 150 and 150 GeV. For $\mu > 0 (< 0)$, the SUSY radiative corrections (in particular, the correction due to sbottom–gluino loops) decrease (increase) substantially the value of m_b, therefore suppressing (enhancing) the $\chi_2^0 \to \chi_1^0 b\bar{b}$ rate by a sizable factor, compared to the branching ratio without the correction (solid lines), for large enough $\tan\beta$ values. The fraction BR($\chi_2^0 \to \chi_1^0 \tau^+ \tau^-$) increases (decreases) then, accordingly. These corrections are therefore very important and must be taken into account.

In Figs. 10, 11 and 12, we show, respectively, the branching fractions BR($\chi_1^+ \to \chi_1^0 \tau^+ \nu_\tau$), BR($\chi_2^0 \to \chi_1^0 \tau^+ \tau^-$) and BR($\chi_2^0 \to \chi_1^0 b\bar{b}$) as functions of μ (> 0) in the models **24, 75, 200** and **OII** without gaugino mass unification as well as in the universal model **1** for comparison. The various parameters are fixed at the following values: $\tan\beta = 50$, $M_2 = 150\,\mathrm{GeV}$, $m_0 = 500$ and $M_A = 100$ (a) and 500 GeV (b). Before discussing the various decay channels in these models, compared to the universal case, let us make two general comments:

(i) The values of $M_{1,2,3}$ at the weak scale are different and modify appreciably the phase space for the decays; in particular two-body decay modes and decays into gluinos become possible. In addition the radiative corrections to

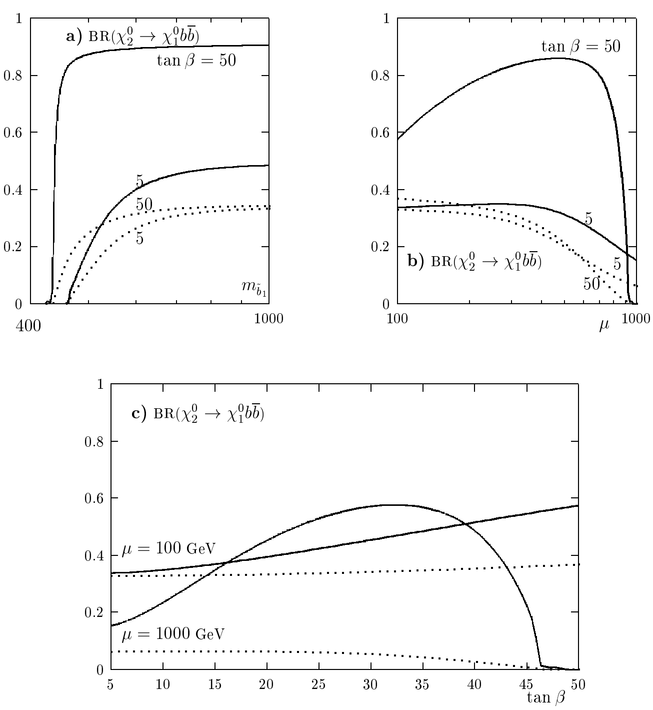

Fig. 8a–c. The branching ratio BR($\chi_2^0 \to \chi_1^0 \bar{b}b$) for two values of $\tan\beta = 5$ and 50 and two values of $M_A = 100\,\mathrm{GeV}$ (solid lines) and 500 GeV (dashed lines) as a function of $m_{\tilde{b}_1}$ for $\mu =$ 500 GeV **a** as a function of μ assuming $m_0 = 300\,\mathrm{GeV}$ **b** and as a function of $\tan\beta$ for two values of $\mu = 100$ and 1000 GeV **c**; M_2 is fixed at 150 GeV

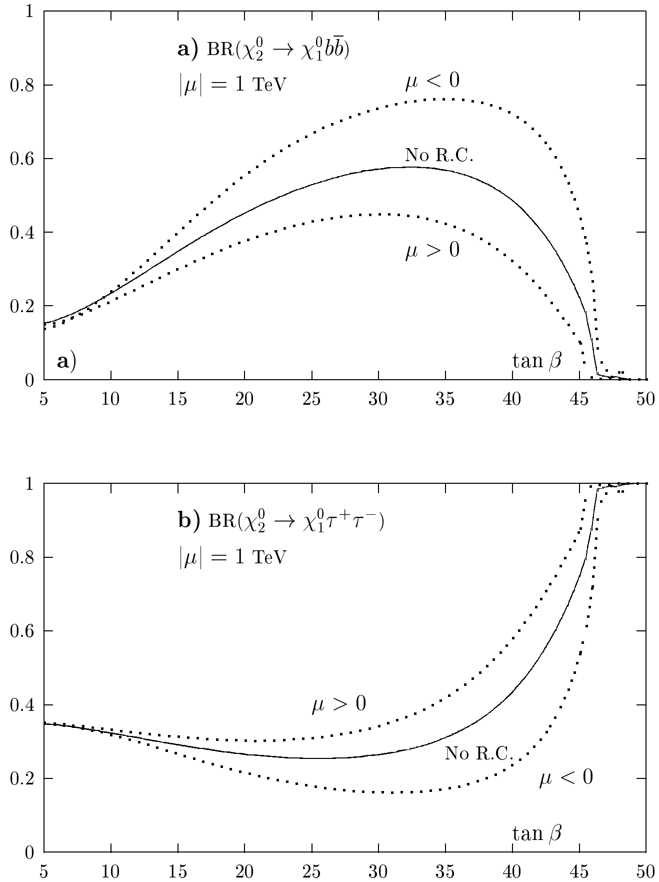

Fig. 9a,b. The branching ratios BR($\chi_2^0 \to \chi_1^0 b\bar{b}$) **a** and BR($\chi_2^0 \to \chi_1^0 \tau^+\tau^-$) **b** as a function of $\tan\beta$ for $|\mu| = 1\,\text{TeV}$, $M_A = 150\,\text{GeV}$, $m_0 = 300\,\text{GeV}$ and $M_2 = 150\,\text{GeV}$, with and without the radiative corrections to the fermion masses

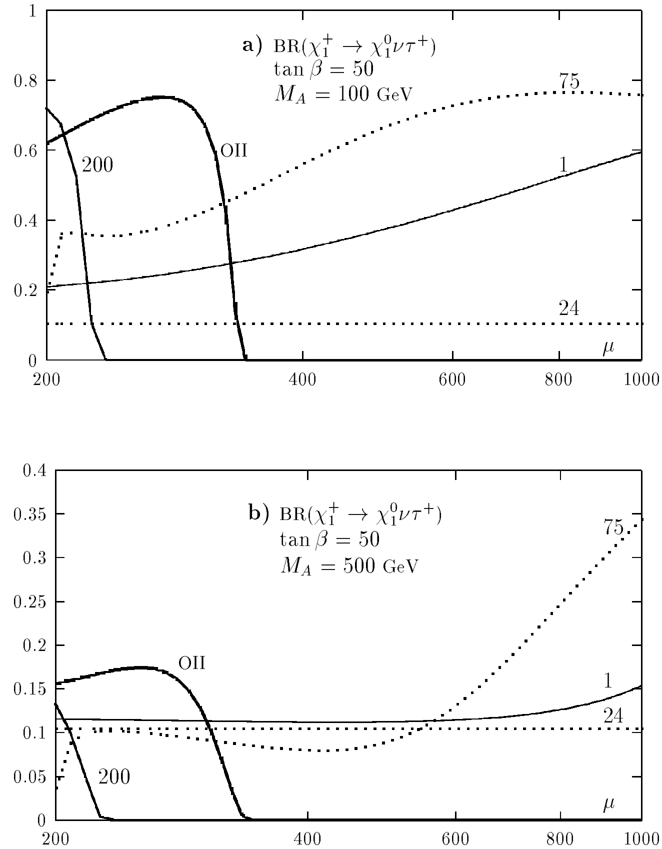

Fig. 10a,b. The branching ratios BR($\chi_1^+ \to \chi_1^0 \nu\tau^+$) as a function of μ in models with non-universal gaugino masses; we have fixed the parameters at $\tan\beta = 50$, $m_0 = 500\,\text{GeV}$, $M_2 = 150\,\text{GeV}$ and $M_A = 100$ (500) GeV for a **b**

the gaugino masses, although only of the order of a few GeV, could allow the opening of channels such as those involving tau leptons.
(ii) Due to the different values of $M_{1,2}$, the evolution of the sfermion masses from Λ_{GUT} to the weak scale are modified, and the contributions of τ-sleptons and bottom squarks can be enhanced or suppressed compared to the universal case. Also, the radiative corrections to the fermion masses are different and can lead to a further enhancement or suppression of the Higgs boson and/or sfermion contribution to the decays.

Model 24: For large μ values, $\mu \gtrsim 200\,\text{GeV}$, the lightest chargino and neutralinos are gaugino-like and because $M_2 \sim 6M_1$, the mass differences $m_{\chi_1^+} - m_{\chi_1^0}$ and $m_{\chi_2^0} - m_{\chi_1^0}$ are large, making the decays into real gauge bosons, $\chi_1^+ \to \chi_1^0 W$ and $\chi_2^0 \to \chi_1^0 Z$, kinematically possible. The branching ratios for χ_1^+ and χ_2^0 are then controlled by the W/Z branching ratios: BR($W \to \tau^+\nu$) $\sim 10\%$, BR($Z \to \tau^+\tau^-$) $\sim 3\%$ and BR($Z \to b\bar{b}$) $\sim 15\%$. For smaller μ values, $\mu \lesssim 200\,\text{GeV}$, the two neutralinos are mixtures of gauginos and higgsinos and three-body decays are possible. The sfermion exchange channels increase the rates for the $\chi_2^0 \to \chi_1^0 \tau^+\tau^-$ and $\chi_1^0 b\bar{b}$ decay channels, with an additional enhancement, in the later channel, being to the exchange of the light Higgs bosons for $M_A, M_h \sim 100\,\text{GeV}$ (this contribution is milder in the case of $\tau^+\tau^-$ final states because of the reduced Yukawa coupling).

Model 75: For $\mu \sim \mathcal{O}(200)\,\text{GeV}$, $m_{\chi_1^+} - m_{\chi_1^0}$ and $m_{\chi_2^0} - m_{\chi_1^0}$ are very small even after the inclusion of the radiative corrections (Fig. 1) and the decays of χ_2^0 and χ_1^+ into the LSP and massive fermions are not kinematically possible (in this case, these particles if they are not almost stable, will decay into the LSP and soft pions). For large values of μ, the mass differences between χ_1^+, χ_2^0 and the LSP are sizable (although penalizing the $b\bar{b}$ final state of χ_2^0) and the different evolution of the sfermion masses as a function of the gaugino masses and the different radiative corrections to the bottom and tau lepton masses, explain the quantitative differences between the branching ratios in the two models **75** and **1**.

Model 200: Here, the chargino χ_1^+ and the LSP are wino-like for large values of μ, and the mass difference $m_{\chi_1^+} - m_{\chi_1^0}$ is too small for the decay $\chi_1^+ \to \chi_1^0 \tau^+\nu$ to occur. For smaller μ values, this decay can receive large contributions from light Higgs bosons and sizable ones from light sfermions (in particular, $\tilde{b}_1$ is lighter than in model **1**). In the case of the decays of the neutralino χ_2^0, since the

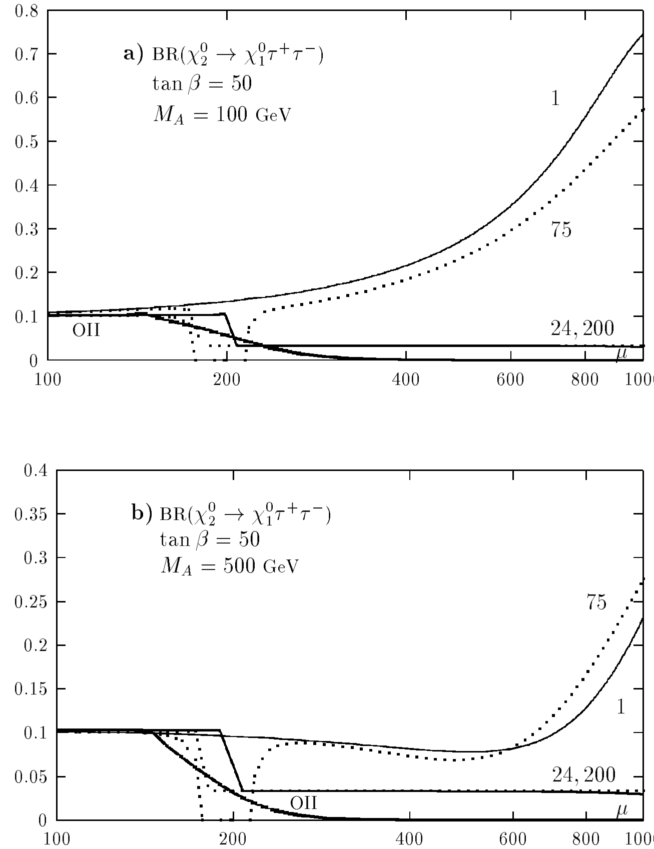

Fig. 11a,b. The branching ratios $\mathrm{BR}(\chi_2^0 \to \chi_1^0\tau^+\tau^-)$ as a function of μ in models with non-universal gaugino masses; we have fixed the parameters at $\tan\beta = 50$, $m_0 = 500\,\mathrm{GeV}$, $M_2 = 150\,\mathrm{GeV}$ and $M_A = 100$ (500) GeV for a **b**

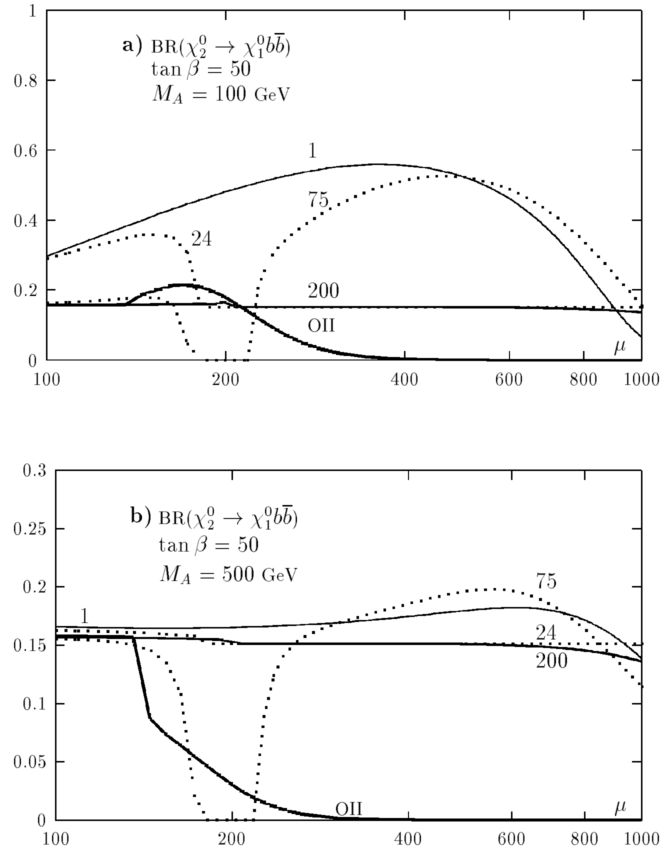

Fig. 12a,b. The branching ratios $\mathrm{BR}(\chi_2^0 \to \chi_1^0 b\bar{b})$ as a function of μ in models with non-universal gaugino masses; we have fixed the parameters at $\tan\beta = 50$, $m_0 = 500\,\mathrm{GeV}$, $M_2 = 150\,\mathrm{GeV}$ and $M_A = 100$ (500) GeV for a **b**

difference $m_{\chi_2^0} - m_{\chi_1^0}$ is always large (exceeding M_Z for $\mu \gtrsim 200\,\mathrm{GeV}$ i.e. when χ_2^0 is bino-like) the branching ratios $\mathrm{BR}(\chi_2^0 \to \chi_1^0\tau^+\tau^-, \chi_1^0 b\bar{b})$ are similar to those of model **24** and are controlled by the Z-boson decay branching ratios. Note that in this scenario, the decay $\chi_2^0 \to \tilde{g}q\bar{q}$ is possible as will be discussed later.

Model OII: In this model, the situation is similar to model **200** for the decays of χ_1^+. Indeed, for $\mu \gtrsim 300\,\mathrm{GeV}$, χ_1^+ is almost degenerate with the LSP and the channel $\chi_1^+ \to \chi_1^0\tau^+\nu$ is kinematically closed. This is almost the case for the neutralino χ_2^0 which has a mass that is close to the LSP mass for large μ values, suppressing the $b\bar{b}$ decay mode. However, the new feature in this scenario is that $M_3 < M_{1,2}$ and for large μ values, the decay modes $\chi_1^+, \chi_2^0, \chi_1^0 \to \tilde{g}q\bar{q}$ open up and become dominant because of the strong interaction part (note, however, that the neutralino χ_1^0 is not the LSP anymore).

Finally, Fig. 13 shows the branching ratio for the decays $\chi_2^0 \to \tilde{g}q\bar{q}$ in the model **200** where $m_{\chi_1^0} < m_{\tilde{g}} < m_{\chi_2^0}$. For small μ values, the lightest neutralinos are higgsino-like and they are degenerate in mass. For values of μ around M_2, the hierarchy $m_{\chi_1^0} < m_{\tilde{g}} < m_{\chi_2^0}$ is possible while $\chi_1^0$is the LSP, and the decay can occur. However, the neutralino couplings to quark–squark pairs are small except in the case of (s)bottoms for large $\tan\beta$ values. In contrast, the χ_1^0–χ_2^0–Z coupling is almost maximal here. $\mathrm{BR}(\chi_2^0 \to \tilde{g}\sum q\bar{q})$, which is approximately the same as $\mathrm{BR}(\chi_2^0 \to \tilde{g}b\bar{b})$, is thus not dominant, but can reach the level of 25%, despite of the fact that it is a mixed strong–electroweak decay mode. For larger values of μ, the neutralinos $\chi_{1,2}^0$ become gaugino-like and the partial decay widths $\Gamma(\chi_2^0 \to \tilde{g}q\bar{q})$ are more important since the couplings to fermion–sfermion pairs are enhanced; however in this case, because $M_3 < M_2$, the gluino becomes lighter than the lightest neutralino which we assume here to be the LSP.

In the case of the charginos, the branching ratio for the decays $\chi_1^+ \to \tilde{g}q\bar{q}'$ for higgsino-like charginos is even smaller, since there is no final state with massive fermions (the $t\bar{b}$ decay mode is not kinematically accessible) and the first and second generation (s)particles have small couplings for higgsino-like charginos. In the gaugino-like region, the lightest chargino becomes lighter than the gluino and the decay does not occur.

We have developed a fortran code called SDECAY [35] which calculates the partial decay widths and branching ratios of the chargino and neutralino decays. It includes not only the three-body decays, $\chi_2^0 \to \chi_1^0 f\bar{f}$ and $\chi_1^+ \to \chi_1^0 f\bar{f}'$ discussed in this paper, but also all the two-

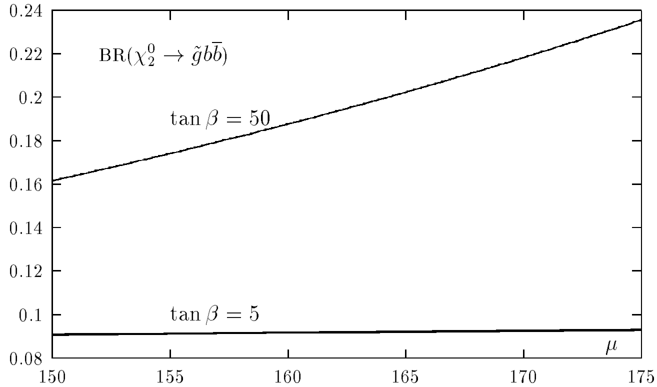

Fig. 13. The branching ratio BR($\chi_2^0 \to \tilde{g} q\bar{q}$) as a function of μ in model **200** with non-universal gaugino masses. The parameters are fixed at $\tan\beta = 5$ and 50, $M_2 = 150\,\mathrm{GeV}$, $m_0 = M_A = 500\,\mathrm{GeV}$

body decays of the charginos and neutralinos (including the heavy $\chi_{3,4}^0$- and χ_2^+-states) into gauge bosons, MSSM Higgs bosons and fermion–sfermion pairs. The program contains, in addition, the branching ratios for the two-, three- and four-body decay modes of the top squarks, as well as the three-body decays of gluinos and all relevant decay modes of sfermions other than the top squarks.

The gaugino mass parameters M_1, M_2, M_3, as well as the soft SUSY-breaking scalar masses $m_{\tilde{f}_L}$ and $m_{\tilde{f}_R}$, can be chosen as free parameters so that decay widths and branching ratios can be obtained in non-universal models. However, scenarios with boundary conditions at high scales are also implemented, since the program has been interfaced with the code SUSPECT [36] for the renormalization-group equations for parameter evolution and for the proper breaking of the electroweak symmetry. For the parameterization of the MSSM Higgs sector, the program has been interfaced with the code HDECAY [37], which in addition gives the decay products for the Higgs particles. All radiative corrections discussed in this analysis are incorporated into the program.

We have compared our results with those of [6] which have been implemented in the program ISAJET [38]. For massless fermions and if the SUSY radiative correction to the fermion masses are not taken into account in the sfermion mass matrices, the agreement was very good in models with gaugino mass unification, giving a great confidence that this rather involved calculation is correct. (The comparison was slightly involved since the evolution of the couplings and the soft SUSY-breaking terms as well as the parameterization of the Higgs sectors are given in different approximations in the programs SUSPECT and ISAJET and we needed to use the same input parameters at low energy in both programs[7].) Our results are however different from those which can be obtained with the program SUSYGEN (version 2.2) [40] used for SUSY particle searches at LEP, since in the latter version, the Higgs boson exchange contributions and the effect of third generation sfermion mixing have not been implemented[8].

5 Conclusions

In this paper, we have analyzed the decay modes of charginos and neutralinos in the MSSM where the lightest neutralino χ_1^0 is the LSP. We focused on the three-body decay modes of the lightest charginos $\chi_1^\pm$ and the next-to-lightest neutralinos χ_2^0 into the LSP and two fermion final states, and made a complete calculation of the decay widths and branching ratios, taking into account all possible channels: vector boson, Higgs boson and sfermion exchange with the mixing in the sfermion sector included. In this context, we have shown that the SUSY radiative corrections to the heavy fermion masses, in particular to the b-quark mass, and to the chargino and neutralino masses can play an important role. We derived full analytical expressions of the Dalitz densities and the integrated partial decay widths in the massless fermion case, and provided the complete formulae for the differential decay widths, including the finite masses of the final fermions and the polarization of the decaying charginos or neutralinos. A fortran code for the numerical evaluation of all the branching ratios is made available [41].

For large values of $\tan\beta$, the bottom and tau Yukawa couplings become large, leading to smaller masses of the tau slepton and bottom squark compared to their first and second generation partners. At the same time, the Yukawa couplings of tau and bottom quarks to the Higgs bosons can become very large. The branching ratios of the decays of the lightest chargino into $\tau\nu$ final states and of the next-to-lightest neutralino into $b\bar{b}$ and $\tau^+\tau^-$ pairs can be thus strongly enhanced in this scenario. We have illustrated this possibility in mSUGRA-type scenarios where the gaugino masses are unified at the GUT scale, but also in scenarios where the boundary conditions for binos and winos are different at this high scale, leading to different mass patterns for the charginos and neutralinos, which affect the decay branching ratios. In particular, new decay channels, such as the decay of the lightest chargino and the next-to-lightest neutralino into gluino and quark–antiquark final states, open up kinematically and can play an important role.

When SUSY particles will decay via cascades through charginos and the heavier neutralinos, the events will contain more τ-leptons and b-quarks, than first and second generation leptons and quarks. This renders the search for SUSY particles and the measurement of the SUSY parameters, where the electron and muon channels where used, less straightforward as already discussed in [7]. b-tagging and the identification of the decays of the tau leptons become then a crucial issue in the search and the study of

[7] We thank Laurent Duflot from ALEPH for his help with this comparison. An independent numerical check in the case of the chargino decays into massless final state fermions, has also been performed by F. Boudjema and V. Lafage [39]

[8] These effects are being included in a new version of the program; we thank S. Katsanevas and N. Ghodbane for discussions on this issue

the properties of these particles, in particular at hadron colliders such as the Tevatron and LHC.

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Appendix

In this appendix, we will give the lengthy formulae for the three-body partial decay widths in the case of finite masses for the fermion final states ($\mu_u \neq \mu_d \neq 0$, with $\mu_f = m_f^2/m_{\chi_i}^2$) and where the polarization of the decaying chargino or neutralino is taken into account[9]:

$$\chi_i(q, n_{\chi_i}) \to \chi_j^0(p) u(p_1) \bar{d}(p_2), \quad \text{(A.1)}$$

where q, p, p_1 and p_2 are the four-momenta of the particles and n_{χ_i} is the spin four-vector of the decaying "ino" defined by $n_{\chi_i} \cdot n_{\chi_i} = -1$ and $n_{\chi_i} \cdot q = 0$.

The partial decay width for both chargino and neutralino three-body decays, following the notation given in Sect. 3, is given by

$$\frac{\mathrm{d}\Gamma_{\chi_i}}{\mathrm{d}\hat{u}\mathrm{d}\hat{t}} = \frac{e^4 m_{\chi_i}}{64(2\pi)^3} N_c \Big[\mathrm{d}\Gamma_V + \mathrm{d}\Gamma_{\tilde{u}} + \mathrm{d}\Gamma_{\tilde{d}} + \mathrm{d}\Gamma_{\Phi} + \mathrm{d}\Gamma_{H_1 H_2} + \mathrm{d}\Gamma_{V\tilde{u}} + \mathrm{d}\Gamma_{V\tilde{d}} + \mathrm{d}\Gamma_{\tilde{u}\tilde{d}} + \mathrm{d}\Gamma_{\Phi\tilde{u}} + \mathrm{d}\Gamma_{\Phi\tilde{d}} \Big], \quad \text{(A.2)}$$

where $\mathrm{d}\Gamma_X$ is decomposed into the spin-independent part (which is half of the unpolarized partial decay width) and the part which depends on the spin four-vector of the decaying "ino":

$$\mathrm{d}\Gamma_X = \frac{1}{2}\mathrm{d}\Gamma_X^U + \mathrm{d}\Gamma_X^S. \quad \text{(A.3)}$$

We will use the reduced Mandelstam variables and spin vector:

$$\hat{u} = (q - p_1)^2/m_{\chi_i}^2, \quad \hat{t} = (q - p_2)^2/m_{\chi_i}^2,$$

and

$$n = n_{\chi_i}/m_{\chi_i}. \quad \text{(A.4)}$$

[9] The expressions for three-body decays of charginos and neutralinos, including the polarization of the initial states, are available in the literature, see [42], in the case of massless final fermions, no Higgs boson exchange and no mixing in the sfermion sector

Spin-independent part

$$\begin{aligned}
\mathrm{d}\Gamma_V^U =& \frac{4}{(1+\mu_\chi+\mu_u+\mu_d-\mu_V-\hat{u}-\hat{t})^2} \\
&\times \Big\{ \left[(G^{\mathrm{L}}_{jiV})^2 + (G^{\mathrm{R}}_{jiV})^2\right] \Big(\Big[(v_V^f)^2 + (a_V^f)^2\Big] \\
&\times \Big[(1+\mu_\chi+\mu_u+\mu_d)(\hat{u}+\hat{t}) - \hat{u}^2 - \hat{t}^2 \\
&- 2\mu_u\mu_d - \mu_u - \mu_d - \mu_\chi(2+\mu_u+\mu_d)\Big] \\
&+ 2\Big[(v_V^f)^2 - (a_V^f)^2\Big]\sqrt{\mu_u\mu_d}[\hat{u}+\hat{t}-\mu_u-\mu_d]\Big) \\
&+ 2\left[(G^{\mathrm{L}}_{jiV})^2 - (G^{\mathrm{R}}_{jiV})^2\right] v_V^f a_V^f \\
&\times [\hat{u}^2 - \hat{t}^2 - (\hat{u}-\hat{t})(1+\mu_\chi+\mu_u+\mu_d) \\
&+ (\mu_d-\mu_u)(1-\mu_\chi)] + 4G^{\mathrm{L}}_{jiV}G^{\mathrm{R}}_{jiV}\sqrt{\mu_\chi} \\
&\times \Big(\Big[(v_V^f)^2 + (a_V^f)^2\Big][\hat{u}+\hat{t}-1-\mu_\chi] \\
&- 4\Big[(v_V^f)^2 - (a_V^f)^2\Big]\sqrt{\mu_u\mu_d}\Big)\Big\}, \quad \text{(A.5)}
\end{aligned}$$

$$\begin{aligned}
\mathrm{d}\Gamma_\Phi^U =& \sum_k \frac{2}{(1+\mu_\chi+\mu_u+\mu_d-\mu_k-\hat{u}-\hat{t})^2} \\
&\times \Big\{ \left[(G^{\mathrm{L}}_{ijk})^2 + (G^{\mathrm{R}}_{ijk})^2\right] \Big([(v_k^f)^2 + (a_k^f)^2] \\
&\times \Big[(1+\mu_\chi+\mu_u+\mu_d)(\hat{u}+\hat{t}) - (\hat{u}+\hat{t})^2 \\
&- (1+\mu_\chi)(\mu_u+\mu_d)\Big] - 2\Big[(v_k^f)^2 - (a_k^f)^2\Big] \\
&\times \sqrt{\mu_u\mu_d}[\hat{u}+\hat{t}-\mu_u-\mu_d]\Big) + 4G^{\mathrm{L}}_{ijk}G^{\mathrm{R}}_{ijk}\sqrt{\mu_\chi} \\
&\times \Big([(v_k^f)^2 + (a_k^f)^2][1+\mu_\chi-\hat{u}-\hat{t}] \\
&- 2\Big[(v_k^f)^2 - (a_k^f)^2\Big]\sqrt{\mu_u\mu_d}\Big)\Big\}, \quad \text{(A.6)}
\end{aligned}$$

$$\begin{aligned}
\mathrm{d}\Gamma_{H_1H_2}^U =& \Big\{ \Big(4v_{H_1}^f v_{H_2}^f\Big) \Big/ \Big((1+\mu_\chi+\mu_u+\mu_d-\mu_{H_1} \\
&- \hat{u}-\hat{t})(1+\mu_\chi+\mu_u+\mu_d-\mu_{H_2}-\hat{u}-\hat{t}) \Big) \Big\} \\
&\times \Big\{ 2\mu_\chi \left[G^{\mathrm{L}}_{ij1}G^{\mathrm{R}}_{ij2} + G^{\mathrm{L}}_{ij2}G^{\mathrm{R}}_{ij1}\right] \\
&\times [1+\mu_\chi-2\sqrt{\mu_u\mu_d}-\hat{u}-\hat{t}] \\
&+ \left[G^{\mathrm{L}}_{ij1}G^{\mathrm{L}}_{ij2} + G^{\mathrm{R}}_{ij1}G^{\mathrm{R}}_{ij2}\right] \Big[(\mu_u+\mu_d) \\
&\times (-1+2\sqrt{\mu_u\mu_d}-\mu_\chi) + (\hat{u}+\hat{t})(1+\mu_\chi \\
&+ \mu_u+\mu_d-2\sqrt{\mu_u\mu_d}) - (\hat{u}+\hat{t})^2\Big]\Big\}, \quad \text{(A.7)}
\end{aligned}$$

$$\begin{aligned}
\mathrm{d}\Gamma_{V\Phi}^U =& \sum_{k=1}^{2} \Big\{ \Big(8\Big) \Big/ \Big((1+\mu_\chi+\mu_u+\mu_d-\mu_{H_k}-\hat{u}-\hat{t}) \\
&\times (1+\mu_\chi+\mu_u+\mu_d-\mu_V-\hat{u}-\hat{t}) \Big) \Big\} \\
&\times \Big\{ \left[G^{\mathrm{L}}_{jiV}G^{\mathrm{R}}_{ijk} + G^{\mathrm{R}}_{jiV}G^{\mathrm{L}}_{ijk}\right] \Big([v_k^f v_V^f + a_k^f a_V^f] \\
&\times \sqrt{\mu_u}(-\mu_\chi-\mu_d+\hat{u})[v_k^f v_V^f - a_k^f a_V^f] \\
&\times \sqrt{\mu_d}(\mu_u+\mu_\chi-\hat{t})\Big) + \left[G^{\mathrm{L}}_{jiV}G^{\mathrm{L}}_{ijk} + G^{\mathrm{R}}_{jiV}G^{\mathrm{R}}_{ijk}\right] \\
&\times \Big([v_k^f v_V^f + a_k^f a_V^f]\sqrt{\mu_u\mu_\chi}(1+\mu_d-\hat{t})
\end{aligned}$$

$$+ [v_k^f v_V^f - a_k^f a_V^f]\sqrt{\mu_d \mu_\chi}(-1 - \mu_u + \hat{u})\Big)\Big\}, \quad \text{(A.8)}$$

$$\begin{aligned} \mathrm{d}\Gamma^U_{\tilde{u}} &= \sum_{k,l=1}^{2} \frac{1}{(-\mu_d - \mu_{\tilde{u}_k} + \hat{t})(-\mu_d - \mu_{\tilde{u}_l} + \hat{t})} \\ &\times \Big\{ - 4a_1^u \sqrt{\mu_\chi}\sqrt{\mu_u \mu_d} + 2a_2^u \sqrt{\mu_\chi \mu_u}(-\mu_d - 1 + \hat{t}) \\ &+ 2a_3^u \sqrt{\mu_d}(-\mu_u - \mu_\chi + \hat{t}) \\ &+ a_4^u \left[-\hat{t}^2 + \hat{t}(1 + \mu_\chi + \mu_d + \mu_u)\right. \\ &- \left.(\mu_\chi + \mu_u)(1 + \mu_d)\right] \Big\}, \end{aligned} \quad \text{(A.9)}$$

$$\begin{aligned} \mathrm{d}\Gamma^U_{\tilde{d}} &= \sum_{k,l=1}^{2} \frac{1}{(-\mu_u - \mu_{\tilde{d}_k} + \hat{u})(-\mu_u - \mu_{\tilde{d}_l} + \hat{u})} \\ &\times \Big\{ - 4a_1^d \sqrt{\mu_\chi}\sqrt{\mu_u \mu_d} + 2a_2^d \sqrt{\mu_\chi \mu_d}(-\mu_u - 1 + \hat{u}) \\ &+ 2a_3^d \sqrt{\mu_u}(-\mu_d - \mu_\chi + \hat{u}) \\ &+ a_4^d[-\hat{u}^2 + \hat{u}(1 + \mu_\chi + \mu_d + \mu_u) \\ &- (\mu_\chi + \mu_d)(1 + \mu_u)] \Big\}, \end{aligned} \quad \text{(A.10)}$$

where

$$\begin{aligned} a_1^f &= (a_{jk}^f b_{jl}^f + a_{jl}^f b_{jk}^f)(a_{ik}^f b_{il}^f + a_{il}^f b_{ik}^f), \\ a_2^f &= (a_{jk}^f b_{jl}^f + a_{jl}^f b_{jk}^f)(a_{ik}^f a_{il}^f + b_{ik}^f b_{il}^f), \\ a_3^f &= (a_{jk}^f a_{jl}^f + b_{jk}^f b_{jl}^f)(a_{ik}^f b_{il}^f + a_{il}^f b_{ik}^f), \\ a_4^f &= (a_{jk}^f a_{jl}^f + b_{jk}^f b_{jl}^f)(a_{ik}^f a_{il}^f + b_{ik}^f b_{il}^f), \end{aligned} \quad \text{(A.11)}$$

$$\begin{aligned} \mathrm{d}\Gamma^U_{V\tilde{d}} &= \sum_{l=1}^{2} \Big\{ \Big(-4\Big) \Big/ \Big((1 + \mu_\chi + \mu_u + \mu_d - \mu_V - \hat{u} - \hat{t}) \\ &\times (-\mu_u - \mu_{\tilde{d}_l} + \hat{u})\Big) \Big\} \Big\{ b_1^f(jiV)[-(\mu_\chi + \mu_d)(\mu_u + 1) \\ &- \hat{u}^2 + \hat{u}(1 + \mu_u + \mu_d + \mu_\chi)] \\ &+ b_2^f(jiV)\sqrt{\mu_\chi}(\hat{u} + \hat{t} - 1 - \mu_\chi) \\ &+ b_3^f(jiV)\sqrt{\mu_u \mu_d}(\hat{u} + \hat{t} - \mu_u - \mu_d) \\ &- 4b_4^f(jiV)\sqrt{\mu_\chi}\sqrt{\mu_u \mu_d} \\ &+ b_5^f(jiV)\sqrt{\mu_d}(\hat{t} - \mu_\chi - \mu_u) \\ &+ b_6^f(jiV)\sqrt{\mu_u \mu_\chi}(\hat{t} - \mu_d - 1) \\ &+ 2b_7^f(jiV)\sqrt{\mu_u}(\hat{u} - \mu_\chi - \mu_d) \\ &+ 2b_8^f(jiV)\sqrt{\mu_\chi \mu_d}(\hat{u} - \mu_u - 1)\Big\}, \end{aligned} \quad \text{(A.12)}$$

$$\begin{aligned} \mathrm{d}\Gamma^U_{V\tilde{u}} &= \sum_{l=1}^{2} \Big\{ \Big(4\Big) \Big/ \Big((1 + \mu_\chi + \mu_u + \mu_d - \mu_V - \hat{u} - \hat{t}) \\ &\times (-\mu_d - \mu_{\tilde{u}_l} + \hat{t})\Big) \Big\} \Big\{ b_1^f(jiV)\sqrt{\mu_\chi}(\hat{u} + \hat{t} - 1 - \mu_\chi) \\ &+ b_2^f(jiV)[-\hat{t}^2 + \hat{t}(1 + \mu_u + \mu_d + \mu_\chi) \\ &- (\mu_\chi + \mu_u)(\mu_d + 1)] - 4b_3^f(jiV)\sqrt{\mu_\chi}\sqrt{\mu_u \mu_d} \\ &+ b_4^f(jiV)\sqrt{\mu_u \mu_d}(\hat{u} + \hat{t} - \mu_u - \mu_d) \\ &+ 2b_5^f(jiV)\sqrt{\mu_u \mu_\chi}(\hat{t} - \mu_d - 1) \\ &+ 2b_6^f(jiV)\sqrt{\mu_d}(\hat{t} - \mu_\chi - \mu_u) \\ &+ b_7^V(jiV)\sqrt{\mu_\chi \mu_d}(\hat{u} - \mu_u - 1) \\ &+ b_8^f(jiV)\sqrt{\mu_u}(\hat{u} - \mu_\chi - \mu_d)\Big\}, \end{aligned} \quad \text{(A.13)}$$

where

$$\begin{aligned} b_1^f(ijk) &= a_{il}^f a_{jl}^f G^{\mathrm{R}}_{ijk}(v_k^f + a_k^f) + b_{il}^f b_{jl}^f G^{\mathrm{L}}_{ijk}(v_k^f - a_k^f), \\ b_2^f(ijk) &= a_{il}^f a_{jl}^f G^{\mathrm{L}}_{ijk}(v_k^f + a_k^f) + b_{il}^f b_{jl}^f G^{\mathrm{R}}_{ijk}(v_k^f - a_k^f), \\ b_3^f(ijk) &= a_{il}^f a_{jl}^f G^{\mathrm{R}}_{ijk}(v_k^f - a_k^f) + b_{il}^f b_{jl}^f G^{\mathrm{L}}_{ijk}(v_k^f + a_k^f), \\ b_4^f(ijk) &= a_{il}^f a_{jl}^f G^{\mathrm{L}}_{ijk}(v_k^f - a_k^f) + b_{il}^f b_{jl}^f G^{\mathrm{R}}_{ijk}(v_k^f + a_k^f), \\ b_5^f(ijk) &= a_{jl}^f b_{il}^f G^{\mathrm{L}}_{ijk}(v_k^f + a_k^f) + a_{il}^f b_{jl}^f G^{\mathrm{R}}_{ijk}(v_k^f - a_k^f), \\ b_6^f(ijk) &= a_{jl}^f b_{il}^f G^{\mathrm{R}}_{ijk}(v_k^f - a_k^f) + a_{il}^f b_{jl}^f G^{\mathrm{L}}_{ijk}(v_k^f + a_k^f), \\ b_7^f(ijk) &= a_{jl}^f b_{il}^f G^{\mathrm{L}}_{ijk}(v_k^f - a_k^f) + a_{il}^f b_{jl}^f G^{\mathrm{R}}_{ijk}(v_k^f + a_k^f), \\ b_8^f(ijk) &= a_{jl}^f b_{il}^f G^{\mathrm{R}}_{ijk}(v_k^f \\ &+ a_k^f) + a_{il}^f b_{jl}^f G^{\mathrm{L}}_{ijk}(v_k^f - a_k^f), \end{aligned} \quad \text{(A.14)}$$

$$\begin{aligned} \mathrm{d}\Gamma^U_{\tilde{u}\tilde{d}} &= \sum_{k,l=1}^{2} \Big\{ \Big(-2\Big) \Big/ \Big((-\mu_u - \mu_{\tilde{d}_l} + \hat{u}) \\ &\times (-\mu_d - \mu_{\tilde{u}_k} + \hat{t})\Big) \Big\} \Big\{ [a_{ik}^u a_{jk}^u b_{il}^d b_{jl}^d + a_{il}^d a_{jl}^d b_{ik}^u b_{jk}^u] \\ &\times \sqrt{\mu_u \mu_d}(\hat{u} + \hat{t} - \mu_u - \mu_d) \\ &+ [a_{ik}^u a_{jk}^u a_{il}^d b_{jl}^d + b_{ik}^u b_{jk}^u b_{il}^d a_{jl}^d]\sqrt{\mu_d}(\hat{t} - \mu_\chi - \mu_u) \\ &+ [a_{jk}^u a_{il}^d a_{jl}^d b_{ik}^u + b_{jk}^u b_{il}^d b_{jl}^d a_{ik}^u]\sqrt{\mu_\chi \mu_d}(\hat{u} - \mu_u - 1) \\ &- 2[a_{jk}^u a_{jl}^d b_{ik}^u b_{il}^d + a_{ik}^u a_{il}^d b_{jk}^u b_{jl}^d]\sqrt{\mu_\chi}\sqrt{\mu_u \mu_d} \\ &+ [a_{jk}^u b_{ik}^u b_{il}^d b_{jl}^d + a_{ik}^u a_{il}^d a_{jl}^d b_{jk}^u]\sqrt{\mu_u}(\hat{u} - \mu_\chi - \mu_d) \\ &+ [a_{jk}^u a_{il}^d b_{ik}^u b_{jl}^d + a_{ik}^u a_{jl}^d b_{jk}^u b_{il}^d](\hat{u}\hat{t} - \mu_\chi - \mu_u \mu_d) \\ &+ [a_{ik}^u a_{jk}^u a_{il}^d a_{jl}^d + b_{ik}^u b_{jk}^u b_{il}^d b_{jl}^d]\sqrt{\mu_\chi}(\hat{u} + \hat{t} - \mu_\chi - 1) \\ &+ [a_{ik}^u a_{jk}^u a_{jl}^d b_{il}^d + b_{ik}^u b_{jk}^u b_{jl}^d a_{il}^d] \\ &\times \sqrt{\mu_\chi \mu_u}(\hat{t} - \mu_d - 1)\Big\}, \end{aligned} \quad \text{(A.15)}$$

$$\begin{aligned} \mathrm{d}\Gamma^U_{\Phi_k \tilde{d}} &= \sum_{k,l} \Big\{ \Big(2\Big) \Big/ \Big((1 + \mu_\chi + \mu_u + \mu_d - \mu_{\Phi_k} - \hat{u} - \hat{t}) \\ &\times (-\mu_u - \mu_{\tilde{d}_l} + \hat{u})\Big) \Big\} \Big\{ b_1^d(ijk)\sqrt{\mu_\chi \mu_u}(\hat{t} - \mu_d - 1) \\ &+ b_2^d(ijk)\sqrt{\mu_u}(-\hat{u} + \mu_\chi + \mu_d) \\ &+ b_3^d(ijk)\sqrt{\mu_\chi \mu_d}(-\hat{u} + \mu_u + 1) \\ &+ b_4^d(ijk)\sqrt{\mu_d}(\hat{t} - \mu_\chi - \mu_u) + b_5^d(ijk) \\ &\times [\hat{u}^2 + \hat{u}\hat{t} - \hat{u}(1 + \mu_u + \mu_d + \mu_\chi) + \mu_u \mu_\chi + \mu_d] \\ &+ 2b_6^d(ijk)\sqrt{\mu_\chi}\sqrt{\mu_u \mu_d} \\ &+ b_7^d(ijk)\sqrt{\mu_u \mu_d}(\hat{u} + \hat{t} - \mu_u - \mu_d) \\ &+ b_8^d(ijk)\sqrt{\mu_\chi}(\hat{u} + \hat{t} - \mu_\chi - 1)\Big\}, \end{aligned} \quad \text{(A.16)}$$

$$\begin{aligned} \mathrm{d}\Gamma^U_{\Phi_k \tilde{u}} &= \sum_{k,l} \Big\{ \Big(2\Big) \Big/ \Big((1 + \mu_\chi + \mu_u + \mu_d - \mu_{\Phi_k} - \hat{u} - \hat{t}) \\ &\times (-\mu_d - \mu_{\tilde{u}_l} + \hat{t})\Big) \Big\} \Big\{ b_1^u(ijk)\sqrt{\mu_u}(\hat{u} - \mu_\chi - \mu_d) \\ &+ b_2^u(ijk)\sqrt{\mu_\chi \mu_u}(-\hat{t} + 1 + \mu_d) \end{aligned}$$

$$+ b_3^u(ijk)\sqrt{\mu_d}(-\hat{t} + \mu_u + \mu_\chi) + b_4^u(ijk)\sqrt{\mu_\chi \mu_d}(\hat{u} - 1 - \mu_u) + 2b_5^u(ijk)\sqrt{\mu_\chi}\sqrt{\mu_u \mu_d} + b_6^u(ijk)[\hat{u}\hat{t} + \hat{t}^2 - \hat{t}(1 + \mu_u + \mu_d + \mu_\chi) + \mu_u + \mu_\chi \mu_d] + b_7^u(ijk)\sqrt{\mu_\chi}(\hat{u} + \hat{t} - \mu_\chi - 1) + b_8^u(ijk)\sqrt{\mu_u \mu_d}(\hat{u} + \hat{t} - \mu_u - \mu_d)\Big\}. \quad (A.17)$$

Spin-dependent part

$$\mathrm{d}\Gamma_V^S = \frac{4}{(1 + \mu_\chi + \mu_u + \mu_d - \mu_V - \hat{u} - \hat{t})^2} \times \Big\{[(G_{jiV}^{\mathrm{L}})^2 - (G_{jiV}^{\mathrm{R}})^2]\Big([(v_V^f)^2 + (a_V^f)^2] \times [p_1.n(\mu_\chi + \mu_d - \hat{u}) + p_2.n(\mu_\chi + \mu_u - \hat{t})] + 2[(v_V^f)^2 - (a_V^f)^2]\sqrt{\mu_u \mu_d}(p_1.n + p_2.n)\Big) + 2\left[(G_{jiV}^{\mathrm{L}})^2 + (G_{jiV}^{\mathrm{R}})^2\right] v_V^f a_V^f \times [-p_1.n(\mu_\chi + \mu_d - \hat{u}) + p_2.n(\mu_\chi + \mu_u - \hat{t})] + 4G_{jiV}^{\mathrm{L}} G_{jiV}^{\mathrm{R}} v_V^f a_V^f \sqrt{\mu_\chi} \times [-p_1.n(1 + \mu_d - \hat{t}) + p_2.n(1 + \mu_u - \hat{u})]\Big\}, \quad (A.18)$$

$$\mathrm{d}\Gamma_\Phi^S = \sum_k \frac{2}{(1 + \mu_\chi + \mu_u + \mu_d - \mu_k - \hat{u} - \hat{t})^2} \times \Big\{[(G_{ijk}^{\mathrm{L}})^2 - (G_{ijk}^{\mathrm{R}})^2][p_1.n + p_2.n] \times \Big([(v_k^f)^2 + (a_k^f)^2][1 + \mu_\chi - \hat{u} - \hat{t}] - 2[(v_k^f)^2 - (a_k^f)^2]\sqrt{\mu_u \mu_d}\Big)\Big\}, \quad (A.19)$$

$$\mathrm{d}\Gamma_{H_1 H_2}^S = \Big\{\Big(4v_{H_1}^f v_{H_2}^f\Big)\Big/\Big((1 + \mu_\chi + \mu_u + \mu_d - \mu_{H_1} - \hat{u} - \hat{t})(1 + \mu_\chi + \mu_u + \mu_d - \mu_{H_2} - \hat{u} - \hat{t})\Big)\Big\} \times [G_{ij1}^{\mathrm{L}} G_{ij2}^{\mathrm{L}} - G_{ij1}^{\mathrm{R}} G_{ij2}^{\mathrm{R}}][p_1.n + p_2.n] \times (1 + \mu_\chi - 2\sqrt{\mu_u \mu_d} - \hat{u} - \hat{t}), \quad (A.20)$$

$$\mathrm{d}\Gamma_{V\Phi}^S = \sum_{k=1}^{2} \Big\{\Big(4\Big)\Big/\Big((1 + \mu_\chi + \mu_u + \mu_d - \mu_{H_k} - \hat{u} - \hat{t}) \times (1 + \mu_\chi + \mu_u + \mu_d - \mu_V - \hat{u} - \hat{t})\Big)\Big\} \times \Big\{[G_{jiV}^{\mathrm{L}} G_{ijk}^{\mathrm{R}} - G_{jiV}^{\mathrm{R}} G_{ijk}^{\mathrm{L}}]\Big([v_k^f v_V^f + a_k^f a_V^f]\sqrt{\mu_u} \times [p_1.n(\hat{t} - 1 - \mu_d) + p_2.n(-\hat{u} - 1 + \mu_u)] + [v_k^f v_V^f - a_k^f a_V^f]\sqrt{\mu_d}[p_1.n(\hat{t} - \mu_d + 1) + p_2.n(-\hat{u} + \mu_u + 1)]\Big) + 2\left[G_{jiV}^{\mathrm{L}} G_{ijk}^{\mathrm{L}} - G_{jiV}^{\mathrm{R}} G_{ijk}^{\mathrm{R}}\right]\sqrt{\mu_\chi} \times \Big(-p_2.n\sqrt{\mu_u}[v_k^f v_V^f + a_k^f a_V^f] + p_1.n\sqrt{\mu_d}[v_k^f v_V^f - a_k^f a_V^f]\Big)\Big\}, \quad (A.21)$$

$$\mathrm{d}\Gamma_{\tilde{u}}^S = \sum_{k,l=1}^{2} \frac{p_2.n}{(-\mu_d - \mu_{\tilde{u}_k} + \hat{t})(-\mu_d - \mu_{\tilde{u}_l} + \hat{t})} \times \left\{2a_{2S}^u\sqrt{\mu_u \mu_\chi} + a_{4S}^u(\hat{t} - \mu_\chi - \mu_u)\right\}, \quad (A.22)$$

$$\mathrm{d}\Gamma_{\tilde{d}}^S = \sum_{k,l=1}^{2} \frac{-p_1.n}{(-\mu_d - \mu_{\tilde{d}_k} + \hat{u})(-\mu_d - \mu_{\tilde{d}_l} + \hat{u})} \times \left\{2a_{2S}^d\sqrt{\mu_d \mu_\chi} + a_{4S}^d(\hat{u} - \mu_\chi - \mu_d)\right\}, \quad (A.23)$$

where

$$a_{2S}^f = (a_{jk}^f b_{jl}^f + a_{jl}^f b_{jk}^f)(a_{ik}^f a_{il}^f - b_{ik}^f b_{il}^f),$$
$$a_{4S}^f = (a_{jk}^f a_{jl}^f + b_{jk}^f b_{jl}^f)(-a_{ik}^f a_{il}^f + b_{ik}^f b_{il}^f), \quad (A.24)$$

$$\mathrm{d}\Gamma_{V\tilde{d}}^S = \sum_{l=1}^{2} \frac{-2}{(1 + \mu_\chi + \mu_u + \mu_d - \mu_V - \hat{u} - \hat{t})(-\mu_u - \mu_{\tilde{d}_l} + \hat{u})} \times \Big\{2b_{1S}^d(jiV)p_1.n(\hat{u} - \mu_\chi - \mu_d) + b_{2S}^d(jiV)\sqrt{\mu_\chi}[p_1.n(\hat{t} - \mu_d - 1) - p_2.n(\hat{u} - \mu_u - 1)] - 2b_{3S}^d(jiV)\sqrt{\mu_u \mu_d}(p_1.n + p_2.n) + b_{5S}^d(jiV)\sqrt{\mu_d}[p_1.n(\hat{t} - \mu_d + 1) - p_2.n(\hat{u} - \mu_u - 1)] - 2b_{6S}^d(jiV)p_2.n\sqrt{\mu_\chi \mu_u} - 4b_{8S}^d(jiV)p_1.n\sqrt{\mu_\chi \mu_d}\Big\}, \quad (A.25)$$

$$\mathrm{d}\Gamma_{V\tilde{u}}^S = \sum_{l=1}^{2} \frac{2}{(1 + \mu_\chi + \mu_u + \mu_d - \mu_V - \hat{u} - \hat{t})(-\mu_d - \mu_{\tilde{u}_l} + \hat{t})} \times \Big\{b_{1S}^u(jiV)\sqrt{\mu_\chi}[p_1.n(\hat{t} - 1 - \mu_d) - p_2.n(\hat{u} - 1 - \mu_u)] + 2b_{2S}^u(jiV)p_2.n(-\hat{t} + \mu_\chi + \mu_u) + 2b_{4S}^u(jiV)\sqrt{\mu_u \mu_d}(p_1.n + p_2.n) + 4b_{5S}^u(jiV)\sqrt{\mu_u \mu_\chi}p_2.n + 2b_{7S}^u(jiV)\sqrt{\mu_\chi \mu_d}p_1.n + b_{8S}^u(jiV)\sqrt{\mu_u} \times [p_1.n(\hat{t} - \mu_d - 1) - p_2.n(\hat{u} - \mu_u + 1)]\Big\}, \quad (A.26)$$

with

$$b_{1S}^f(ijk) = a_{il}^f a_{jl}^f G_{ijk}^{\mathrm{R}}(v_k^f + a_k^f) - b_{il}^f b_{jl}^f G_{ijk}^{\mathrm{L}}(v_k^f - a_k^f),$$
$$b_{2S}^f(ijk) = a_{il}^f a_{jl}^f G_{ijk}^{\mathrm{L}}(v_k^f + a_k^f) - b_{il}^f b_{jl}^f G_{ijk}^{\mathrm{R}}(v_k^f - a_k^f),$$
$$b_{3S}^f(ijk) = a_{il}^f a_{jl}^f G_{ijk}^{\mathrm{R}}(v_k^f - a_k^f) - b_{il}^f b_{jl}^f G_{ijk}^{\mathrm{L}}(v_k^f + a_k^f),$$
$$b_{4S}^f(ijk) = a_{il}^f a_{jl}^f G_{ijk}^{\mathrm{L}}(v_k^f - a_k^f) - b_{il}^f b_{jl}^f G_{ijk}^{\mathrm{R}}(v_k^f + a_k^f),$$
$$b_{5S}^f(ijk) = a_{jl}^f b_{il}^f G_{ijk}^{\mathrm{L}}(v_k^f + a_k^f) - a_{il}^f b_{jl}^f G_{ijk}^{\mathrm{R}}(v_k^f - a_k^f),$$
$$b_{6S}^f(ijk) = a_{jl}^f b_{il}^f G_{ijk}^{\mathrm{R}}(v_k^f - a_k^f) - a_{il}^f b_{jl}^f G_{ijk}^{\mathrm{L}}(v_k^f + a_k^f),$$
$$b_{7S}^f(ijk) = a_{jl}^f b_{il}^f G_{ijk}^{\mathrm{L}}(v_k^f - a_k^f) - a_{il}^f b_{jl}^f G_{ijk}^{\mathrm{R}}(v_k^f + a_k^f),$$
$$b_{8S}^f(ijk) = a_{jl}^f b_{il}^f G_{ijk}^{\mathrm{R}}(v_k^f + a_k^f) - a_{il}^f b_{jl}^f G_{ijk}^{\mathrm{L}}(v_k^f - a_k^f), \quad (A.27)$$

$$\mathrm{d}\Gamma^S_{\tilde{u}\tilde{d}} = \sum_{k,l=1}^{2} \frac{-1}{(-\mu_u - \mu_{\tilde{d}_l} + \hat{u})(-\mu_d - \mu_{\tilde{u}_k} + \hat{t})} \times\Big\{ 2[a^u_{ik}a^u_{jk}b^d_{il}b^d_{jl} - a^d_{il}a^d_{jl}b^u_{ik}b^u_{jk}]\sqrt{\mu_u\mu_d} \times[p_1.n + p_2.n] +[a^u_{ik}a^u_{jk}a^d_{il}b^d_{jl} - b^u_{ik}b^u_{jk}b^d_{il}a^d_{jl}]\sqrt{\mu_d}[p_1.n(\hat{t} - \mu_d + 1) -p_2.n(\hat{u} - \mu_u - 1)] -2[a^u_{jk}a^d_{il}a^d_{jl}b^u_{ik} - b^u_{jk}b^d_{il}b^d_{jl}a^u_{ik}]\sqrt{\mu_\chi\mu_d}p_1.n +[a^u_{jk}b^u_{ik}b^d_{il}b^d_{jl} - a^u_{ik}a^d_{il}a^d_{jl}b^u_{jk}]\sqrt{\mu_u} \times[p_1.n(-\hat{t} + \mu_d + 1) - p_2.n(-\hat{u} + \mu_u - 1)] +[a^u_{jk}a^d_{il}b^u_{ik}b^d_{jl} - a^u_{ik}a^d_{jl}b^u_{jk}b^d_{il}] \times[p_1.n(-\hat{t} - \mu_d + 1) + p_2.n(-\hat{u} - \mu_u + 1) +[a^u_{ik}a^u_{jk}a^d_{il}a^d_{jl} - b^u_{ik}b^u_{jk}b^d_{il}b^d_{jl}]\sqrt{\mu_\chi} \times[p_1.n(\hat{t} - \mu_d - 1) - p_2.n(\hat{u} - \mu_u - 1)] +2[a^u_{ik}a^u_{jk}a^d_{jl}b^d_{il} - b^u_{ik}b^u_{jk}b^d_{jl}a^d_{il}] \times\sqrt{\mu_\chi\mu_u}p_2.n\Big\}, \quad (A.28)$$

$$\mathrm{d}\Gamma^S_{\Phi_k\tilde{d}} = \sum_{k,l} \frac{1}{(1 + \mu_\chi + \mu_u + \mu_d - \mu_{\Phi_k} - \hat{u} - \hat{t})(-\mu_u - \mu_{\tilde{d}_l} + \hat{u})} \times\Big\{ - 2b^d_{1S}(ijk)\sqrt{\mu_\chi\mu_u}p_2.n +b^d_{2S}(ijk)\sqrt{\mu_u}[p_1.n(\hat{t} - \mu_d - 1) + p_2.n(-\hat{u} + \mu_u - 1)] +2b^d_{3S}(ijk)\sqrt{\mu_\chi\mu_d}p_1.n +b^d_{4S}(ijk)\sqrt{\mu_d}[p_1.n(\hat{t} - \mu_d + 1) + p_2.n(-\hat{u} + \mu_u + 1)] +b^d_{5S}(ijk) \times[p_1.n(2\hat{u} + \hat{t} - 2\mu_\chi - \mu_d - 1) + p_2.n(\hat{u} + \mu_u - 1)] +2b^d_{7S}(ijk)\sqrt{\mu_u\mu_d}[p_1.n + p_2.n] +b^d_{8S}(ijk) \times\sqrt{\mu_\chi}[p_1.n(-\hat{t} + \mu_d + 1) + p_2.n(\hat{u} - \mu_u - 1)]\Big\}, \quad (A.29)$$

$$\mathrm{d}\Gamma^S_{\Phi_k\tilde{u}} = \sum_{k,l} \frac{1}{(1 + \mu_\chi + \mu_u + \mu_d - \mu_{\Phi_k} - \hat{u} - \hat{t})(-\mu_d - \mu_{\tilde{u}_l} + \hat{t})} \times\Big\{b^u_{1S}(ijk) \sqrt{\mu_u}[p_1.n(\hat{t} - \mu_d - 1) + p_2.n(-\hat{u} + \mu_u - 1)] -2b^u_{2S}(ijk)\sqrt{\mu_\chi\mu_u}p_2.n +b^u_{3S}(ijk)\sqrt{\mu_d}[p_1.n(\hat{t} - \mu_d + 1) + p_2.n(-\hat{u} + \mu_u + 1)] +2b^u_{4S}(ijk)\sqrt{\mu_\chi\mu_d}p_1.n +b^u_{6S}(ijk) \times[p_1.n(-\hat{t} - \mu_d + 1) + p_2.n(-\hat{u} - 2\hat{t} + 2\mu_\chi + \mu_u + 1)] +b^u_7(ijk)\sqrt{\mu_\chi}[p_1.n(-\hat{t} + \mu_d + 1) + p_2.n(\hat{u} - \mu_u - 1)] -2b^d_{8S}(ijk)\sqrt{\mu_u\mu_d}[p_1.n + p_2.n]\Big\}. \quad (A.30)$$

Phase space

To obtain the integrated partial widths, one has to express $\hat{u}$ and $\hat{t}$ as functions of x_1 and x_2

$$\hat{u} = 1 - x_1 + \mu_u, \quad \hat{t} = 1 - x_2 + \mu_d, \quad (A.31)$$

and integrate over the latter variables, with boundary conditions:

$$2\sqrt{\mu_u} \leq x_1 \leq 1 + [\mu_u - (\sqrt{\mu_d} + \sqrt{\mu_\chi})^2], \quad (A.32)$$

$$s_{\min} \leq x_2 \leq s_{\max}, \quad (A.33)$$

$$s_{\min} = \frac{1}{2}\frac{(x_1 - 2)(x_1 - 1 - \mu_d + \mu_\chi - \mu_u) - \sqrt{\Delta}}{1 - x_1 + \mu_u},$$
$$s_{\max} = \frac{1}{2}\frac{(x_1 - 2)(x_1 - 1 - \mu_d + \mu_\chi - \mu_u) + \sqrt{\Delta}}{1 - x_1 + \mu_u}, \quad (A.34)$$

with

$$\Delta = (\mu_u - x_1^2)\left[\frac{1}{4}\mu_d\mu_\chi - (x_1 - 1 + \mu_d + \mu_\chi - \mu_u)^2\right]. \quad (A.35)$$

References

1. For reviews on the MSSM, see: P. Fayet, S. Ferrara, Phys. Rep. **32**, 249 (1977); H.P. Nilles, Phys. Rep. **110**, 1 (1984); R. Barbieri, Riv. Nuov. Cim. **11**, 1 (1988); R. Arnowitt, Pran Nath, Report CTP-TAMU-52-93; M. Drees, S. Martin, CLTP Report (1995) and hep-ph/9504324; S. Martin, hep-ph/9709356; J. Bagger, Lectures at TASI-95, hep-ph/9604232
2. H.E. Haber, G. Kane, Phys. Rep. **117**, 75 (1985)
3. P. Fayet, Phys. Lett. B **69**, 489 (1977)
4. A.H Chamseddine, R. Arnowitt, P. Nath, Phys. Rev. Lett. **49**, 970 (1982); R. Barbieri, S. Ferrara, C.A Savoy, Phys. Lett. B **119**, 343 (1982); L. Hall, J. Lykken, S. Weinberg, Phys. Rev. D **27**, 2359 (1983); for a recent review, see: M. Drees, S. Martin in [1]
5. See e.g: A. Bartl, H. Fraas, W. Majerotto, Nucl. Phys. B **278**, 1 (1986); for reviews and a complete list of references, see for instance: G.F. Giudice et al., Report of the Searches for New Physics working group in Physics at LEP2, Vol. 1, p. 463–524, hep-ph/9602207; S. Abel et al., Report of the SUGRA working group for RUNII at the Tevatron, hep-ph/0003154
6. H. Baer, C.H. Chen, M. Drees, F. Paige, X. Tata, Phys. Rev. D **58**, 075008 (1998)
7. H. Baer, C.H. Chen, M. Drees, F. Paige, X. Tata, Phys. Rev. D **59**, 055014 (1999); Phys. Rev. Lett. **79**, 986 (1997)
8. A. Bartl, W. Majerotto, W. Porod, Phys. Lett. B **465**, 187 (1999)
9. A. Djouadi, Y. Mambrini, Phys. Lett. B **493**, 120 (2000) (hep-ph/0007174)
10. See for instance: V. Barger, M.S. Berger, R.J. Phillips, Phys. Lett. B **314**, 351 (1993); M. Carena, S. Pokorski, C. Wagner, Nucl. Phys. B **406**, 59 (1993)

11. P. Igo–Kemenes, Status of the Higgs boson searches, LEPC meeting on November 3, http://lephiggs.web.cern.ch/LEPHIGGS/talks/index.html; ALEPH Collaboration (R. Barate et al.), Phys. Lett. B **495**, 1 (2000); L3 Collaboration (M. Acciarri et al.), Phys. Lett. B **495**, 18 (2000); DELPHI Collaboration (P. Abreu et al.), Phys. Lett. B **499**, 23 (2001); PAL Collaboration (G. Abbiendi et al.), Phys. Lett. B **499**, 38 (2001)
12. A. Brignole, C.E. Ibánez, C. Munoz, Nucl. Phys. B **422**, 125 (1995); P. Binetruy, M.K. Gaillard, B.D. Nelson, hep-ph/0011081
13. L. Randall, R. Sundrum, Nucl. Phys. B **557**, 79 (1999); G. Giudice, M. Luty, H. Murayama, R. Rattazzi, JHEP **9812**, 027 (1998)
14. C.H. Chen, M. Drees, J.F. Gunion, Phys. Rev. D **55**, 330 (1997); (E) ibid. D **60**, 039901 (1999); J. Amundson et al., Report of the Snowmass Supersymmetry Theory Working Group, hep-ph/9609374
15. C. Boehm, A. Djouadi, Y. Mambrini, Phys. Rev. D **61**, 095006 (2000); A. Djouadi, Y. Mambrini, hep-ph/0011364 (Phys. Rev. D, to appear)
16. J.F. Gunion, H.E. Haber, Nucl. Phys. B **272**, 1 (1986); (E) hep-ph/9301205
17. M. El Kheishen, A. Shafik, A. Aboshousha, Phys. Rev. D **45**, 4345 (1992)
18. A. Djouadi, J. Kalinowski, P. Ohmann, P.M. Zerwas, Z. Phys. C **74**, 93 (1997); see also, Report of the Higgs working group in the ECFA–DESY Workshop (1996) edited by P.M. Zerwas et al., hep-ph/9605437
19. See, e.g., S. Komine, M. Yamguchi, hep-ph/0007327
20. D. Pierce, A. Papadopoulos, Nucl. Phys. B **430**, 278 (1994); Phys. Rev. D **50**, 565 (1994); A.B. Lahanas, K. Tamkavis, N.D. Tracas, Phys. Lett. B **324**, 387 (1994)
21. D. Pierce, J. Bagger, K. Matchev, R.J. Zhang, Nucl. Phys. B **491**, 3 (1997)
22. M. Drees, M.M. Nojiri, D.P. Roy, Y. Yamada, Phys. Rev. D **56**, 276 (1997)
23. G. Passarino, M. Veltman, Nucl. Phys. B **160**, 151 (1979)
24. For relatively recent work see: C.H. Chen, M. Drees, J.F. Gunion, Phys. Rev. Lett. **76**, 2002 (1996); G.F. Giudice, A. Pomarol, Phys. Lett. B **372**, 253 (1996); A. Datta, S. Maity, Phys. Rev. D **59**, 055019 (1999); J. Gunion, S. Mrenna, hep-ph/0103167
25. J. Ellis, S. Rudaz, Phys. Lett. B **128**, 248 (1983); M. Drees, K. Hikasa, Phys. Lett. B **252**, 127 (1990)
26. L. Hall, R. Rattazzi, U. Sarid, Phys. Rev. D **50**, 7048 (1994); R. Hempfling, Z. Phys. C **63**, 309 (1994); M. Carena, M. Olechowski, S. Pokorski, C. Wagner, Nucl. Phys. B **426**, 269 (1994); F. Borzumati, M. Olechowski, S. Pokorski, Phys. Lett. B **349**, 311 (1995); A. Donini, Nucl. Phys. B **467**, 3 (1996)
27. N. Gray, D.J. Broadhurst, W. Grafe, K. Schilcher, Z. Phys. C **48**, 673 (1990)
28. For a review on the Higgs sector of the MSSM, see, e.g., J. Gunion, H. Haber, G. Kane, S. Dawson, The Higgs hunter's guide (Addison–Wesley, Reading 1990); A. Djouadi, Int. J. Mod. Phys. A **10**, 1 (1995); M. Spira, Fortsch. Phys. **46**, 203 (1998)
29. Y. Okada, M. Yamaguchi, T. Yanagida, Prog. Theor. Phys. **85**, 1 (1991); H. Haber, R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991); J. Ellis, G. Ridolfi, F. Zwirner, Phys. Lett. B **257**, 83 (1991); R. Barbieri, F. Caravaglios, M. Frigeni, Phys. Lett. B **258**, 167 (1991); M. Drees, M.M. Nojiri, Phys. Rev. D **45**, 2482 (1992); R. Hempfling, A. Hoang, Phys. Lett. B **331**, 99 (1994); M. Carena, M. Quiros, C.E.M. Wagner, Nucl. Phys. B **461**, 407 (1996); H. Haber, R. Hempfling, A. Hoang, Z. Phys. C **75**, 539 (1997); S. Heinemeyer, W. Hollik, G. Weiglein, Phys. Rev. D **58**, 091701 (1998); J.R. Espinosa, R.J. Zhang, JHEP **0003**, 026 (2000); Nucl. Phys. B **586**, 3 (2000); M. Carena et al., Nucl. Phys. B **580**, 29 (2000)
30. M. Carena, J.R. Espinosa, M. Quiros, C.E.M. Wagner, Phys. Lett. B **335**, 209 (1995)
31. A. Bartl, W. Majerotto, W. Porod, Z. Phys. C **64**, 499 (1994); (E) C **68**, 515 (1995)
32. A. Denner, H. Eck, O. Hahn, J. Kublbeck, Nucl. Phys. B **387**, 467 (1992)
33. J.F. Gunion, H.E. Haber, Phys. Rev. D **37**, 2515 (1988); J.F. Gunion et al., Int. J. Mod. Phys. A **2**, 1035 (1987); A **2**, 1145 (1987)
34. Particle Data Group (D.E. Groom et al.), Eur. Phys. J. C **15**, 1 (2000); for a recent compilation of LEP2 results, see the talks of S. Rosier–Lees at the LEPC meeting on November 3, 2000, of J.F. Grivaz at the Conference Higgs and SUSY, Orsay, March 2001
35. Y. Mambrini, SDECAY: a program for the decays of SUSY particles, to be released
36. A. Djouadi, J.L. Kneur, G. Moultaka, SUSPECT: a program for the supersymmetric spectrum, hep-ph/9901246
37. A. Djouadi, J. Kalinowski, M. Spira, Comput. Phys. Commun. **108**, 56 (1998)
38. H. Baer, F.E. Paige S.D. Protopopescu, X. Tata, ISAJET 7.48: A Monte-Carlo event generator for $pp, p\bar{p}$ and e^+e^- reactions, hep-ph/0001086
39. See the notes of F. Boudjema, V. Lafage, L. Duflot, S. Rosier–Lees, in Report of the GDR-Supersymétrie (to appear)
40. S. Katsanevas, P. Morawitz, Comput. Phys. Commun. **112**, 227 (1998)
41. Y. Mambrini, PhD thesis, University of Montpellier II (to appear)
42. Discussions on polarized decays of charginos and neutralinos for massless final state fermions can be found e.g. in: M. Chen, C. Dionisi, M. Martinez, X. Tata, Phys. Rep. **159**, 201 (1988); J.L. Feng, M.J. Strassler, Phys. Rev. D **55**, 1326 (1997); G. Moortgat-Pick et al., Eur. Phys. J. C **7**, 113 (1999); Eur. Phys. J. C **9**, 521 (1999); S.Y. Choi et al., Eur. Phys. J. C **7**, 123 (1999); Eur. Phys. J. C **8**, 669 (1999); Eur. Phys. J. C **14**, 535 (2000); S.Y. Choi, H.S. Song, W.Y. Song, Phys. Rev. D **61**, 075004 (2000)